PERFORMANCE ANALYSIS OF THREE STAGE MANPOWER MODEL

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ABSTRACT

A manpower organization may deal with various recruitment policies for its employees for the better grade of service and welfare of the employees. For this purpose a certain process is required so that proper appointments may be ensured. In this investigation matrix approach has been used for a manpower model having three-stage structure. The steady state behavior has been analyzed by finding out the mean number of employees at each stage. The joint probability distribution for the system size and the limiting behavior has been examined. The numerical experiment has also been performed to provide sensitivity analysis.

Keywords: Queueing network, Manpower planning, Promotions, Wastage and mobility, Matrix method.

INTRODUCTION

In any business house or industry manpower planning is of importance. For better working, the management has to keep some incentive to the employees or they should be promoted to the higher positions in their service. There are some established methods to control manpower system namely (I) recruitment (II) promotions and (III) wastage and mobility. The human resource department in any organization looks after these phenomena and needs objective method to control the planning of personnel. It is evident that human behavior is random, therefore stochastic models can be helpful in providing the efficient design and analysis of manpower system. Many authors have suggested the control of wastage by labor turn over in their study while others considered force of mobility. Srinivasan and Saavitri (2002) facilitated a cost analysis on univariate policies of recruitment in manpower models. Srinivasan and Saavitri (2003) also proposed a cost analysis using bivariate policies of recruitment in manpower planning. Sharma and Ahuja (2004) investigated the serial and parallel queueing networks in managing promotional advancement policy. Such models are often encountered in real life situations. Mengqiao et al. (2016) discussed a data-driven approach to manpower planning at U.S.–Canada border crossings. The review article on application of queueing theory in health care proposed Lakshmi and Sivakumar (2013).

In this paper we attempt to find the solution of a manpower-planning model with three-graded system with the help of matrix method. We consider employee's requirement as well as promotion processes at the three stages. The incoming and leaving processes are considered to be Poisson. The performance measures to compute the average number of employees in each stage and in the system are established. The algorithm to evaluate the steady state probability vector is outlined.

MODEL DESCRIPTION

We consider three stages service commission model via matrix method, which has been employed by considering stages 1, 2 and 3 respectively. Employee arrives at stage 1 according to a Poisson process with mean rate λ; after getting service from stage 1, the employee moves to next stage 2 with Poisson parameter Λ_0 and leaves stage 1, without promotion with rate α. The employee is promoted from stage 2 to stage 3 according to Poisson process with parameter δ_0 and leaves stage 2 without promotion with rate β. The employee leaves stage 3 with parameter γ.

The transition flow is depicted with the help of schematic diagram as shown in figure 1.

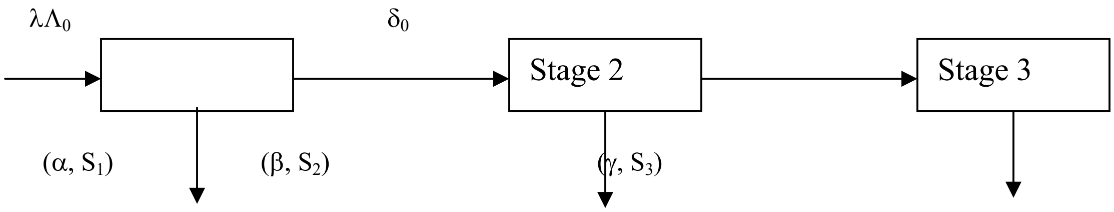

Figure 1: Schematic diagram for three stage service model

We develop stochastic model based upon the following assumptions -

- The probability that one employee enters in grade 1 during the interval $(t, t+\Delta t)$ is $\lambda\Delta t$.
- The probability that the employee leaves the grade 1, when there are S_1 employees in grade 1 during the interval $(t, t+\Delta t)$ is $(\alpha S_1)\Delta t$.
- The probability that the employee got the promotion from grade 1 to grade 2, when there are S_1 employees in grade 1 during the interval $(t, t+\Delta t)$ is $(\Lambda_0 S_1)\Delta t$.
- The probability that the employee got the promotion from grade 2 to grade 3, when there are S_2 employees in grade 2 during the interval $(t, t+\Delta t)$ is $(\delta_0 S_2)\Delta t$.
- The probability that the employee leaves the grade 2, when there are S_2 employees in grade 2 in an interval $(t, t+\Delta t)$ is $(\beta S_2)\Delta t$.
- The probability that the employee leaves the grade 3, when there are S_3 employees in grade 3 in an interval $(t, t+\Delta t)$ is $(\gamma S_3)\Delta t$.
- The probability that no employee joined any grade, no employee got promotion and no one leaves from any grade during interval $(t, t+\Delta t)$ is$[1 - (\lambda + \alpha S_1 + \Lambda_0 S_1 + \delta_0 S_2 + \beta S_2 + \gamma S_3)\Delta t]$.
- The maximum numbers of employees allowed in stage 1, 2 and 3 are N, M and K, respectively.
- Let $P_{(S_1,S_2,S_3)}$ denotes the steady stateprobability that there are S_1 employees in stage 1, S_2 employees in stage 2, and S_3 employees in stage 3.

The difference - differential equations governing the model are

$$0 = \lambda P_{(S_1-1,S_2,S_3)} + \alpha(S_1+1)P_{(S_1+1,S_2,S_3)} + \Lambda_0(S_1+1)P_{(S_1+1,S_2-1,S_3)} + \beta(S_2+1)P_{(S_1,S_2+1,S_3)}$$
$$+ \delta_0(S_2+1)P_{(S_1,S_2+1,S_3-1)} + \gamma(S_3+1)P_{(S_1,S_2,S_3+1)} - (\lambda + \alpha S_1 + \Lambda_0 S_1 + \beta S_2 + \delta_0 S_2 + \gamma S_3)P_{(S_1,S_2,S_3)}$$
$$1 \le S_1 \le N, 1 \le S_2 \le M, 1 \le S_3 \le K$$

(1)

$$0 = \lambda P_{(S_1-1,S_2,0)} + \alpha(S_1+1)P_{(S_1+1,S_2,0)} + \Lambda_0(S_1+1)P_{(S_1+1,S_2-1,0)}$$
$$+ \beta(S_2+1)P_{(S_1,S_2+1,0)} + \gamma P_{(S_1,S_2,1)} - (\lambda + \alpha S_1 + \Lambda_0 S_1 + \beta S_2 + \delta_0 S_2)P_{(S_1,S_2,0)} \quad (2)$$
$$1 \le S_1 \le N, 1 \le S_2 \le M$$

$$0 = \lambda P_{(S_1-1,0,S_3)} + \alpha(S_1+1)P_{(S_1+1,0,S_3)} + \beta P_{(S_1,1,S_3)} + \delta_0 P_{(S_1,1,S_3-1)}$$
$$+ \gamma(S_3+1)P_{(S_1,0,S_3+1)} - (\lambda + \alpha S_1 + \Lambda_0 S_1 + \gamma S_3)P_{(S_1,0,S_3)}\ ;\ 1 \le S_1 \le N, 1 \le S_3 \le K \quad (3)$$

$$0 = \alpha P_{(1,S_2,S_3)} + \Lambda_0 P_{(1,S_2-1,S_3)} + \beta(S_2+1)P_{(0,S_2+1,S_3)} + \delta_0(S_2+1)P_{(0,S_2+1,S_3-1)}$$
$$+ \gamma(S_3+1)P_{(0,S_2,S_3+1)} - (\lambda + \beta S_2 + \delta_0 S_2 + \gamma S_3)P_{(0,S_2,S_3)}\ ;\ 1 \le S_2 \le M, 1 \le S_3 \le K \quad (4)$$

$$0 = \lambda P_{(S_1-1,0,0)} + \alpha(S_1+1)P_{(S_1+1,0,0)} + \beta P_{(S_1,1,0)} + \gamma P_{(S_1,0,1)} - (\lambda + \alpha S_1 + \Lambda_0 S_1)P_{(S_1,0,0)}\ ;1 \le S_1 \le N \quad (5)$$

$$0 = \alpha P_{(1,S_2,0)} + \Lambda_0 P_{(1,S_2-1,0)} + \beta(S_2+1)P_{(0,S_2+1,0)} + \gamma P_{(0,S_2,1)} - (\lambda + \beta S_2 + \delta_0 S_2)P_{(0,S_2,0)}\ ;1 \le S_2 \le M \quad (6)$$

$$0 = \alpha P_{(1,0,S_3)} + \beta P_{(0,1,S_3)} + \delta_0 P_{(0,1,S_3-1)} + \gamma(S_3+1)P_{(0,0,S_3+1)} - (\alpha + \gamma S_3)P_{(0,0,S_3)}\ ;\ 1 \le S_3 \le K \quad (7)$$

$$0 = \alpha P_{(1,0,0)} + \beta P_{(0,1,0)} + \gamma P_{(0,0,1)} - \lambda P_{(0,0,0)} \quad (8)$$

THE ANALYSIS

The above equations (1)-(8) can be put in the matrix form Q P = 0 with P as steady state probability vector and Q as given below

$$Q = \begin{bmatrix} A_0 & B_0 & 0 & \ldots\ldots & 0 & 0 \\ C & A_1 & 2B_0 & \ldots\ldots & 0 & 0 \\ 0 & C & A_2 & \ldots\ldots & 0 & 0 \\ .. & .. & .. & \ldots\ldots & .. & .. \\ .. & .. & .. & \ldots\ldots & .. & .. \\ 0 & 0 & 0 & \ldots\ldots & A_{(N-1)} & NB_0 \\ 0 & 0 & 0 & \ldots\ldots & C & A_N \end{bmatrix} \text{ where, } A_0 = \begin{bmatrix} A_{00} & B_0 & 0 & \ldots\ldots & 0 & 0 \\ 0 & A_{01} & 2B_0 & \ldots\ldots & 0 & 0 \\ 0 & 0 & A_{02} & \ldots\ldots & 0 & 0 \\ .. & .. & .. & \ldots\ldots & .. & .. \\ .. & .. & .. & \ldots\ldots & .. & .. \\ 0 & 0 & 0 & \ldots\ldots & A_{0(M-1)} & MB_{00} \\ 0 & 0 & 0 & \ldots\ldots & 0 & A_{0M} \end{bmatrix}$$

$$B_0 = \begin{bmatrix} \alpha & 0 & 0 & \ldots & 0 & 0 \\ 0 & \alpha & 0 & \ldots & 0 & 0 \\ \Lambda_0 & 0 & \alpha & \ldots & 0 & 0 \\ .. & .. & .. & \ldots & .. & .. \\ .. & .. & .. & \ldots & .. & .. \\ 0 & 0 & \Lambda_0 & \ldots & 0 & \alpha \end{bmatrix}, \; C = \begin{bmatrix} \lambda & 0 & 0 & \ldots & 0 & 0 \\ 0 & \lambda & 0 & \ldots & 0 & 0 \\ .. & .. & .. & \ldots & .. & .. \\ .. & .. & .. & \ldots & .. & .. \\ 0 & 0 & 0 & \ldots & \lambda & 0 \\ 0 & 0 & 0 & \ldots & 0 & \lambda \end{bmatrix}; \; A_{00} = \begin{bmatrix} -\lambda & \gamma & 0 & 0 & \ldots & 0 \\ 0 & -(\lambda+\gamma) & 0 & 0 & \ldots & 0 \\ 0 & 0 & -(\lambda+\gamma) & 0 & \ldots & 0 \\ .. & .. & .. & .. & \ldots & .. \\ .. & .. & .. & .. & \ldots & .. \\ 0 & 0 & 0 & 0 & \ldots & -(\lambda+K\gamma) \end{bmatrix},$$

$$A_{0j} = \begin{bmatrix} -\begin{pmatrix} \lambda + j\delta_0 \\ +j\beta \end{pmatrix} & \gamma & 0 & \ldots\ldots & 0 & 0 \\ 0 & -\begin{pmatrix} \lambda + j\delta_0 \\ +j\beta+\gamma \end{pmatrix} & 2\gamma & \ldots\ldots & 0 & 0 \\ .. & .. & .. & \ldots\ldots & .. & .. \\ .. & .. & .. & \ldots\ldots & .. & .. \\ 0 & 0 & 0 & \ldots\ldots & -\begin{pmatrix} \lambda + j\delta_0 + j\beta \\ +(K-1)\gamma \end{pmatrix} & K\gamma \\ 0 & 0 & 0 & \ldots\ldots & 0 & \begin{pmatrix} \lambda + j\delta_0 \\ +j\beta+K\gamma \end{pmatrix} \end{bmatrix}; \quad j=1, 2, \ldots\ldots, M \; ; \; B_{00} = \begin{bmatrix} \delta_0 & 0 & 0 & \ldots & 0 & 0 \\ \beta & \delta_0 & 0 & \ldots & 0 & 0 \\ 0 & \beta & \delta_0 & \ldots & 0 & 0 \\ .. & .. & .. & \ldots & .. & .. \\ .. & .. & .. & \ldots & .. & .. \\ 0 & 0 & 0 & \ldots & \beta & \delta_0 \end{bmatrix}$$

$$, \; A_{00}' = \begin{bmatrix} -\left(\lambda+\alpha+\Lambda_0\right) & \gamma & 0 & \ldots\ldots & 0 & 0 \\ 0 & -\begin{pmatrix} \lambda+\alpha \\ +\Lambda_0+\gamma \end{pmatrix} & 2\gamma & \ldots\ldots & 0 & 0 \\ .. & .. & .. & \ldots\ldots & .. & .. \\ .. & .. & .. & \ldots\ldots & .. & .. \\ 0 & 0 & 0 & \ldots\ldots & -\begin{pmatrix} \lambda+\alpha+\Lambda_0 \\ +(K-1)\gamma \end{pmatrix} & K\gamma \\ 0 & 0 & 0 & \ldots\ldots & 0 & -\begin{pmatrix} \lambda+\alpha+\Lambda_0 \\ +K\gamma \end{pmatrix} \end{bmatrix}$$

$$A_N = \begin{bmatrix} A'_{0K} & B_{00} & 0 & \ldots\ldots & 0 & 0 \\ 0 & A'_{1K} & 2B_{00} & \ldots\ldots & 0 & 0 \\ 0 & 0 & A'_{2K} & \ldots\ldots & 0 & 0 \\ .. & .. & .. & \ldots\ldots & .. & .. \\ .. & .. & .. & \ldots\ldots & .. & .. \\ 0 & 0 & 0 & \ldots\ldots & A'_{(M-1)} & MB_{00} \\ 0 & 0 & 0 & \ldots\ldots & 0 & A'_{MK} \end{bmatrix}; \quad A_1 = \begin{bmatrix} A'_{00} & B_{00} & 0 & \ldots\ldots & 0 & 0 \\ 0 & A'_{1K} & 2B_{00} & \ldots\ldots & 0 & 0 \\ 0 & 0 & A'_{2K} & \ldots\ldots & 0 & 0 \\ .. & .. & .. & \ldots\ldots & .. & .. \\ .. & .. & .. & \ldots\ldots & .. & .. \\ 0 & 0 & 0 & \ldots\ldots & A'_{(M-1)K} & MB_{00} \\ 0 & 0 & 0 & \ldots\ldots & 0 & A'_{MK} \end{bmatrix}$$

$$A'_{jK} = \begin{bmatrix} -\begin{pmatrix} \lambda+\alpha+\Lambda_0 \\ +j\delta_0+j\beta \end{pmatrix} & \gamma & 0 & \ldots.. & 0 & 0 \\ 0 & -\begin{pmatrix} \lambda+\alpha+\Lambda_0 \\ +j\delta_0+j\beta \\ +\gamma \end{pmatrix} & 2\gamma & \ldots.. & 0 & 0 \\ .. & .. & .. & \ldots.. & .. & .. \\ .. & .. & .. & \ldots.. & .. & .. \\ 0 & 0 & 0 & \ldots.. & -\begin{pmatrix} \lambda+\alpha+\Lambda_0 \\ +j\delta_0+j\beta \\ +(K-1)\gamma \end{pmatrix} & K\gamma \\ 0 & 0 & 0 & \ldots.. & 0 & -\begin{pmatrix} \lambda+\alpha+\Lambda_0 \\ +j\delta_0+j\beta \\ +K\gamma \end{pmatrix} \end{bmatrix}; \quad j = 1, 2, \ldots.., M$$

where

$$A''_{0K} = \begin{bmatrix} -\begin{pmatrix} \lambda+i\alpha \\ +i\Lambda_0 \end{pmatrix} & \gamma & 0 & \ldots\ldots & 0 & 0 \\ 0 & -\begin{pmatrix} \lambda+i\alpha \\ +i\Lambda_0 \\ +\gamma \end{pmatrix} & 2\gamma & \ldots\ldots & 0 & 0 \\ .. & .. & .. & \ldots\ldots & .. & .. \\ .. & .. & .. & \ldots\ldots & .. & .. \\ 0 & 0 & 0 & \ldots\ldots & -\begin{pmatrix} \lambda+i\alpha \\ +i\Lambda_0 \\ +(M-1)\gamma \end{pmatrix} & M\gamma \\ 0 & 0 & 0 & \ldots\ldots & 0 & -\begin{pmatrix} \lambda+i\alpha+i\Lambda_0 \\ +M\gamma \end{pmatrix} \end{bmatrix}; \quad i = 1, 2, \ldots\ldots N$$

The steady state vector P associated with Q such that P Q = 0, can be written in partition form along with the normalizing condition P e = 1, where e is a column vector of appropriate dimension with all the elements equal to 1.
Let us partition P as P = (P_0, P_1, P_2,…,P_N)
Where, P_0 = (P_{000}, P_{001}, ….,P_{00K}, P_{010}, P_{011}, ….,P_{01K}, P_{020}, P_{021},….,P_{0MK})
P_1 = (P_{100}, P_{101}, P_{102},….,P_{10K}, P_{110}, P_{111},….,P_{11K}, P_{120}, P_{121},….,P_{1MK})
P_N = (P_{N00}, P_{N01},….,P_{N0K}, P_{N10}, P_{N11},….,P_{N1K}, P_{N20} ,P_{N21},….,P_{NMK})
Now we shall provide an algorithm to compute steady state distribution vector P.
The linear equations are given by:

$$P_0 A_0 + P_1 C = 0 \ ; nP_{n-1}B_0 + P_n A_n + P_{n+1}C = 0 \quad ; n = 1, 2, \ldots\ldots.. N-1 \ ; (9)$$

$$NP_{(N-1)}B_0 + P_N A_N = 0 \ (10)$$

Using equations (10) we can express P_i' s (i =1, 2,…..,N-1) in term of P_0 as $P_k = P_0 Z(k)$; k =1, 2, ……, N where $Z(1) = -A_0 C^{-1}$; $Z(2) = -\left[A_1 Z(1) + B_0\right] C^{-1}$ $Z(k+1) = -\left[A_k Z(k) + kB_0 Z(k-1)\right] C^{-1}$; $k = 2, 3, \ldots\ldots, N-1$

$Z(N) = -\left[A_{N-1} Z(N-1) + (N-1) B_0 Z(N-2)\right] C^{-1}$.

SOME PERFORMANCE CHARACTERISTICES

Using probabilities, we can establish various performance measures as follows:

- The expected number of employees in the system is given by

$$E(Q) = \sum_{S_1=1}^{N} \sum_{S_2=1}^{M} \sum_{S_3=1}^{K} (S_1 + S_2 + S_3) P_{(S_1, S_2, S_3)} \tag{11}$$

- The average number of employees in grade 1 is

$$E(N_1) = \sum_{S_1=1}^{N} \sum_{S_2=1}^{M} \sum_{S_3=1}^{K} S_1 P_{(S_1, S_2, S_3)} \tag{12}$$

- The average number of employees in grade 2 is

$$E(N_2) = \sum_{S_1=1}^{N} \sum_{S_2=1}^{M} \sum_{S_3=1}^{K} S_2 P_{(S_1, S_2, S_3)} \tag{13}$$

- The average number of employees in grade 3 is

$$E(N_3) = \sum_{S_1=1}^{N} \sum_{S_2=1}^{M} \sum_{S_3=1}^{K} S_3 P_{(S_1, S_2, S_3)} \tag{14}$$

NUMERICAL RESULTS

In this section, numerical results for the average number of employees and expected number of employees in the system are calculated using MATLAB software and are summarized in tables 1and 2. For illustration purpose, we get N = 5, M = 5, K = 5.Table 1 depicts the results for the number of employees and expected jobs in the queue for the different values of (α, β, γ) and fixed value for (λ, Λ_0, δ_0) as (5.5, 3.5, 0.1). We see that the number of employees increases in each stage with rates α, β and γ.

Table 1: Average number of employees in each stage and in the system for different values of (α, β, γ)

α	β	γ	**E(N_1)**	**E(N_2)**	**E(N_3)**	**E(Q)**
0.1	0.2	0.1	3.39	3.75	3.43	10.57
0.3	0.2	0.1	3.41	3.76	3.45	10.62
0.5	0.2	0.1	3.43	3.78	3.46	10.67
0.7	0.2	0.1	3.44	3.78	3.48	10.70
0.2	0.3	0.1	3.47	3.86	3.49	10.82
0.2	0.5	0.1	3.66	4.09	3.65	11.40
0.2	0.7	0.1	3.87	4.32	3.82	12.01
0.2	0.9	0.1	4.06	4.53	4.01	12.60
0.2	0.2	0.3	3.43	3.79	3.46	10.68
0.2	0.2	0.5	3.45	3.82	3.48	10.75
0.2	0.2	0.7	3.48	3.85	3.51	10.84
0.2	0.2	0.9	3.51	3.88	3.53	10.92

Table 2: Average number of employees in each stage and in the system for different parameters (λ , Λ_0 , δ_0)

λ	Λ_0	δ_0	**E(N_1)**	**E(N_2)**	**E(N_3)**	**E(Q)**
3.0	2.0	0.5	4.33	4.75	4.29	13.37
4.0	2.0	0.5	4.04	4.60	4.05	12.69
5.0	2.0	0.5	3.85	4.56	3.93	12.35
6.0	2.0	0.5	3.74	4.56	3.88	12.18
4.5	1.0	0.5	4.07	4.91	4.32	13.30
4.5	1.5	0.5	4.00	4.76	4.13	12.89
4.5	2.0	0.5	3.93	4.57	3.98	12.48
4.5	2.5	0.5	3.89	4.41	3.89	12.19
4.5	3.5	0.2	3.62	3.93	3.80	11.35
4.5	3.5	0.4	3.79	4.13	3.76	11.68
4.5	3.5	0.6	4.00	4.37	3.95	12.32
4.5	3.5	0.8	4.22	4.60	4.16	12.98

In table 2, we display the average number of employees in the queue for fixed value of (α, β, γ) as (0.1, 0.2, 0.3) and different value of (λ, Λ_0, δ_0). The value of E(N_i), i = 1, 2, 3 and E(Q) decrease as λ, Λ_0 increase. However there is reverse effect of δ_0 as expected number of employees in each stage and E(Q) increases with the increment in δ_0.

DISCUSSION

In manpower organization, the exit or wastage of personnel is very common. Wastage of personnel due to retirement, death and resignations is a common phenomenon in administrative as well as production-oriented organization. In this paper we have described a manpower model with the assumption that the organization is having three-stage structure. The difference differential equations are constructed to examine the steady state behavior of the system. Numerical results are obtained for illustration purpose, which demonstrate the validity of analytical results.

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