# EXTREMAL SETS WITH MIN MAX PROPERTIES

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**ABSTRACT**

The authors have introduced a new class of sets on a topological space which was named as class of extremal sets [P. Agarwal and C.K. Goel, 2017] and studied various properties of these extremal sets in terms of subsets, supersets, intersection and union of sets. In the present paper, the authors continue the study of these sets and prove various characteristics and properties of these sets in the presence of min max properties. Further it has also been studied how extremal sets behave in the presence of a topological space.

**Keywords : E**xtremal Set, Extremal points.

## INTRODUCTION

A variety of sets has been defined on a topological space by various mathematicians since when the topology was introduced on a non – empty set X such as θ-open sets, $\alpha$-opensets, preopen sets,$\omega_\theta$-open sets, ω–open sets to name a few. Some of these sets behaved in an eccentric manner. These sets have been proved very useful in the study of topological spaces and have extensively been used in the study of generalized covering and separation axioms. Maintaining the spirit of generalizing open sets Nakaoka and Oda in 2001 introduced minimal open sets[F. Nakaoka and N. Oda, 2001], maximal open sets[F. Nakaoka and N. Oda, 2003], compliments to which were given the notions of minimal closed sets and maximal closed sets[F. Nakaoka and N. Oda, 2006]. These sets have been defined as follows :

Let (X, $\Im$) be a topological space then a proper nonempty open set U of X is called a minimal open set when there does not exist any open subsets of X lying between U and ∅. Similarly, a nonempty open set U of X is called a maximal open set when the only open supersets of U are U and X. In a similar fashion, we can define minimal / maximal closed subsets of X.

Taking clue from these sets, the authors have introduced a new class of sets on a topological space in [P. Agarwal and C.K. Goel, 2017]. These sets have beennamed as extremal sets and have been defined as follows :

A subset A⊆X is said to be an extremal set if for everyx∈ A and for every y ∈X-A there exists a proper open set in X containing both x and y.

It can be easily seen that in real line with usual topology, any subset of X is an extremal set.Similarly in a topological space X with discrete topology, every subset of X is anextremal set whereas in a topological space X with indiscrete topology, no subset of X is an extremalset. As an another example in point inclusion topology, every subset A of X is an extremal set asa∈A andx ∈X-A there exists a proper open set {p,a,x} in X containing both a and x where p is the point of inclusion.

Properties of these extremal sets have been studied in terms of subsets, supersets, intersection and union of sets. It has been found that extremal sets and open sets are in general independent to each other. However in the presence of both the properties i.e. a set being both open and extremal, new characterizations of such sets arisewhich have also been studied in [P. Agarwal and C.K. Goel, 2017].

In the present paper wecontinue the study of these sets and prove various characteristics and properties of these sets in the presence of min max properties.
As usual, for any subset A of a topological space (X,$\Im$ ), we denote by cl(A) [resp. int(A)], closure of A [interior of A]. Further, let O(X) denote the collection of all open subsets of X. We study the behavior of maximal and minimal open sets along with extremal sets in the presence of certain topological properties.

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Some of the results proved are:
**Theorm.** If U is a minimal open and extremal set then U is contained in at least one neighborhood of each element of X.
**Proof.** Let $x \in X$. If $x \in U$ then U is the neighborhood of x. If $x \in X- U$ then since U is extremal, there exists a neighborhood V of x such that $U \cap V \neq \emptyset$. Further, since $U \cap V \subseteq U$ and U is minimal hence we must have $U \subseteq V$.

**Theorm.**If U is a maximal open and extremal set then U contains complement of at least oneneighborhood of each element of X-U.
**Proof.** Let $x \in X- U$ then since U is an extremal set, there exists a neighborhood $U_x$of x suchthat $U \cap U_x \neq \emptyset$. But since U is maximal, $U_x \cup U = X \Rightarrow (X-U_x) \subseteq U$.

If A is an extremal set which is maximal open then, there exists an open set containing all of X-A.
These sets have been applied in the study of connected topological spaces and it has been observed that:

**Theorm.** A maximal open set in a connected set is not an extremal set.
**Theorm.** If A is a maximal open set and U is an open set in X then either U or X-U isproperly contained in A.

Further the concepts of min max properties, connectedness and extremal sets when studied together produced some interesting results such as :

**Theorm.**In a connected space, if A is a maximal open set then, every open set in Xintersects A.

**Proof.** Let X be a connected space and let U be an open set in X such that $A \cap U = \varphi$. Theneither U= X-A in which case A and X-A will form an open partition of X making X disconnected contrary to our assumption or $U \subseteq X-A$ in which case $U \cup A$ will be a proper open set containing A contrary to the maximality of A. Hence U must intersect A.

**Theorm.**In a connected space a maximal open set is dense.

**Proof.** Let X be a connected space and let A be a maximal open subset of X. we show that X-cl(A) = φ since X-cl(A) is an open set which is disjoint of A we have $A \cup (X\text{-cl}(A)) = X$. Hence A and X- cl(A) form a separation of X contradicting the connectivity of X. Therefore we must have X- cl(A) = φ implying that cl(A) = X. Hence A is dense in X.

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