SEARCHING IMPROVED EXPONENTIAL RATIO TYPE ESTIMATORS OF POPULATION MEAN

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ABSTRACT

In this study, two efficient estimators of the population mean using known population median of study variable have been proposed. The proposed estimators are compared with existing concerned ratio estimators of the population mean using auxiliary information. It can be seen from the table of numerical analysis that the proposed ratio estimator is best among the class of all mentioned existing ratio type estimators of the population mean of study variable.
Keywords: Main variable, Exponential ratio estimators, Bias, mean squared error, Efficiency.

INTRODUCTION

Sampling is necessary whenever the population is very large. The most appropriate estimator for estimating any parameter is the corresponding statistics. Thus most appropriate estimator is the sample mean for the population mean. But the weakness of this estimator is that the sampling distribution of this estimator is very much dispersed. Our aim is to find the improved estimator for the population mean. It is done through the use of auxiliary information collected on the additional cost of the survey. On the other hand, without increasing the cost of the survey, we can use known population parameter of study variable such as population median for improved estimation. Ratio, product and regression type estimators are used for improved estimation of population mean under different situations.

All above estimators including ratio, product and regression type estimators make use of auxiliary information, and the drawback with the use of auxiliary information is that it is collected on the additional cost of the survey. Thus the alternative is to use some known parameter of study variable easily available and improves the efficiency of the estimator. There are so many examples where median is easily available, Subramani (2016). In the present paper, we have used population median of study variable for improved estimation of population mean of the variable under study.

LITERATURE REVIEW

Over the years, several estimators of the population mean such as the naive unbiased sample mean estimator, usual regression estimator, usual ratio estimator and modified ratio type estimators have been proposed in the literature. Many authors have given various estimators of the population mean using auxiliary information for improved estimation. A summary of the literature review of some existing estimators is presented in Table 1.

Table 1: Various estimators along with their description

S.No.	Estimator	Description
1.	$t_o = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$	An unbiased estimator of population mean of study variable without using auxiliary variable.
2.	$t_1 = \bar{y} + \beta(\bar{X} - \bar{x})$ Watson (1937)	Make use of auxiliary information in the form of population mean and sample mean of auxiliary variable for improved estimation of population mean of study variable y when the study and auxiliary variables are highly positively correlated to each other and the line of regression y on x passes through origin.

3.	$t_2 = \bar{y}\dfrac{\bar{X}}{\bar{x}}$ Cochran (1940)	This estimator also makes use of auxiliary information for improved estimation of population mean of study variable when the line of regression y on x does not pass through origin and both variables are highly correlated to each other.
4.	$t_3 = \bar{y}\exp\left[\dfrac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]$ Bahl and Tuteja (1991)	Exponential ratio type estimator using above auxiliary information which makes use of same positively correlated auxiliary information as the usual ratio estimator does.
5.	$t_4 = \bar{y}\left(\dfrac{\bar{x}}{\bar{X}}\right)^{\alpha}$ Srivastava (1967)	This estimator is the generalization of ratio estimator which makes use of positively correlated auxiliary information in the form of population mean and sample mean of auxiliary variable.
6.	$t_5 = \bar{y}\left[\dfrac{\bar{X}}{\bar{X}+\alpha(\bar{x}-\bar{X})}\right]$ Reddy (1974)	This general ratio type estimator also makes use of positively correlated auxiliary information for improved estimation of population mean.
7.	$t_6 = \bar{y}\left(\dfrac{\bar{x}}{\bar{X}}\right)^{\delta}\exp\left(\dfrac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$ Kadilar (2016)	This generalized ratio-cum-exponential ratio type estimator makes use of highly positively correlated auxiliary variable for improved estimation of population mean of study variable.
8.	$t_7 = \bar{y}\dfrac{M}{m}$ Subramani (2016)	Makes use of population median of study variable for improved estimation of population mean of study variable without increasing the cost of the survey.

PROPOSED ESTIMATORS

We propose two new exponential ratio type estimators of population mean using known population median of study variable as,

$$t_{p_1} = \bar{y}\left(\frac{m}{M}\right)\exp\left[\frac{a(M-m)}{(M+m)}\right] \tag{1}$$

$$t_{p_2} = \bar{y}\left(\frac{m}{M}\right)\exp\left[\frac{(M-m)}{a(M+m)}\right] \tag{2}$$

Where, a is a suitably chosen constant to be determined such that the MSEs of the proposed estimators t_{p_i} ($i = 1,2$) is minimum.

The biases, mean squared errors and minimum mean squared errors of these proposed estimators, up to the approximation of degree one are respectively given by,

$$B(t_{p_1}) = \bar{Y}\left[\frac{1-f}{n}\left(\frac{a^2}{8}-\frac{a}{4}\right)C_m^2 + \frac{1-f}{n}\left(1-\frac{a}{2}\right)C_{ym} + \left(1-\frac{a}{2}\right)\frac{Bias(m)}{M}\right]$$

$$MSE(t_{p_1}) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + a_1^2C_m^2 + 2a_1C_{ym}] \text{ ,, where, } a_1 = \left(1-\frac{a}{2}\right)$$

$$MSE_{\min}(t_{p_1}) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 - \frac{C_{ym}^2}{C_m^2}\right] \text{ for } a_{1opt} = -C_{ym}/C_m^2 \tag{3}$$

$$B(t_{p_2}) = \bar{Y}\left[\frac{1-f}{n}\left(\frac{1}{8a^2}-\frac{1}{4a}\right)C_m^2 + \frac{1-f}{n}\left(1-\frac{1}{2a}\right)C_{ym} + \left(1-\frac{1}{2a}\right)\frac{Bias(m)}{M}\right]$$

$$MSE(t_{p_2}) = \frac{1-f}{n}\bar{Y}^2[C_y^2 + a_2^2 C_m^2 + 2a_2 C_{ym}], \text{ where, } a_2 = \left(1-\frac{1}{2a}\right) \quad (4)$$

$$MSE_{\min}(t_{p_2}) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2 - \frac{C_{ym}^2}{C_m^2}\right] \text{ for } a_{2opt} = -C_{ym}/C_m^2 \quad (5)$$

Thus, we see from (3) and (5) that the minimum mean squared errors of both the proposed estimators t_{p_1} and t_{p_2} are the same.

EFFICIENCY COMPARISON

The proposed estimators are better than the usual mean per unit estimator, usual regression estimator by Watson (1937), Usual ratio estimator by Cochran (1940), Bahl and Tuteja (1991) exponential ratio type estimator, Srivastava (1967) generalized ratio type estimator, Reddy (1974) ratio type estimator, Kadilar (2016) exponential ratio type estimator and Subramani (2016) ratio type estimator of population mean respectively under the following conditions:

$$V(t_0) - MSE_{\min}(t_{p_i}) > 0, (i = 1,2), \text{ if } \frac{C_{ym}^2}{C_m^2} > 0, \text{ or if } C_{ym}^2 > 0$$

$$MSE(t_1) - MSE_{\min}(t_{p_i}) > 0, \text{ if } \frac{C_{ym}^2}{C_m^2} - C_y^2\rho_{yx}^2 > 0$$

$$MSE(t_2) - MSE_{\min}(t_{p_i}) > 0, \text{ if } C_x^2 - 2C_{yx} + \frac{C_{ym}^2}{C_m^2} > 0, \text{ or } C_x^2 + \frac{C_{ym}^2}{C_m^2} > 2C_{yx}$$

$$MSE(t_3) - MSE_{\min}(t_{p_i}) > 0, \text{ if } \frac{C_x^2}{4} - C_{yx} + \frac{C_{ym}^2}{C_m^2} > 0, \text{ or } \frac{C_x^2}{4} + \frac{C_{ym}^2}{C_m^2} > C_{yx}$$

$$MSE(t_4) - MSE_{\min}(t_{p_i}) > 0, \text{ if } \frac{C_{ym}^2}{C_m^2} - C_y^2\rho_{yx}^2 > 0$$

Under the same above condition, proposed estimators t_{p_i} are also better than t_5 and t_6.

$$MSE(t_7) - MSE_{\min}(t_{p_i}) > 0, \text{ if } R_7^2 C_m^2 - 2R_7 C_{ym} + \frac{C_{ym}^2}{C_m^2} > 0, \text{ or } R_7^2 C_m^2 + \frac{C_{ym}^2}{C_m^2} > 2R_7 C_{ym}$$

NUMERICAL EXAMPLE

To justify the theoretical findings of proposed estimators along with other mentioned estimators of the population mean, we have considered a natural population is given in Subramani (2016). Following Tables 2 and 3 represent the parameter values for the population along with constants and mean squared errors of existing and proposed estimators respectively.

Table 2: Parameter values and constants for the natural population	
Parameter	Population
N	34
n	5
${}^{N}C_{n}$	278256
$\bar{Y}$	856.4118
$\bar{M}$	736.9811
M	767.5
$\bar{X}$	208.8824
R_7	1.1158
C_y^2	0.125014
C_x^2	0.088563
C_m^2	0.100833
C_{ym}	0.07314
C_{yx}	0.047257
ρ_{yx}	0.4491

Table 3: Mean squared error of various estimators	
Estimator	Population
t_0	15640.97
t_1	12486.75
t_2	14895.27
t_3	12498.01
t_4	12486.75
t_5	12486.75
t_6	12486.75
t_7	10926.53
$t_{p_i}\,(i=1,2)$	9001.92

RESULT ANALYSIS

It can be seen from Table 3 that the proposed estimators are better than all mentioned competing estimators of the population mean of study variable. Thus proposed estimators may be used for the improved estimation of population mean without increasing the cost of the survey.

CONCLUSION

This research deals with the estimation of population mean of study variable using known population median of study variable. The expressions for the biases and mean squared errors for both the estimators have been derived up to the approximation of degree one. The proposed estimators are compared theoretically with the other mentioned existing estimators of the population mean, which make use of auxiliary information, collected on the additional cost of the survey. Theoretical findings are numerically justified using three natural populations by Subramani (2016).

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