SOLUTION OF FUZZY GAME PROBLEM USING TRIDECAGONAL FUZZY NUMBER

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ABSTRACT

An approach to solve a fuzzy game problem using a new fuzzy number has been considered in this paper. A tridecagonal fuzzy number is used as imprecise values in the fuzzy game matrix. We shall solve the fuzzy game problem by using ranking of tridecagonal fuzzy number. By ranking, we shall convert the fuzzy valued game problem to a crisp valued game problem , which we shall solve using standard method.

Keywords: *Fuzzy Number, Tridecagonal Fuzzy Number, Fuzzy Arithmetic, Ranking Of Fuzzy Number, Fuzzy Game Problem*.

INTRODUCTION

(Zadeh 1965) introduced the fuzzy set theory. It provides a normal way of handling problems in which the point of indistinctness and impression occurs. In decision analysis, the ambiguity existing in input information is generally denoted as fuzzy numbers (Kumar & Gupta, 2010). The maximization and minimization of fuzzy set, uncertainty and information was introduced by Chen (1985). The arithmetic operations, alpha cut, and ranking function are already introduced for existing fuzzy number by (Liou & Wang, 1992; Chen & Klein, 1997).

In recent literatures many types of fuzzy numbers like triangular, trapezoidal, pentagonal, hexagonal, heptagonal, octagonal, etc have been introduced with their membership function. We have also seen many applications and various operations of these fuzzy numbers. Here we have solved the fuzzy game problem using Tridecagoanal fuzzy number also known as Triskaidecagonal fuzzy number (Rajkumar & Helen, 2018). A ranking function was introduced by (Yager, 1986). We convert the fuzzy valued game problem into a crisp valued problem by using the ranking functionand then solve the problem using the standard method.

TRIDECAGONAL FUZZY NUMBER

A Tridecagoanal fuzzy number is denoted as $\bar{A}_{TD}$= (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}) and its membership function is given by:

$$\mu_{\tilde{A}_{TD}}(x) = \begin{cases} \frac{1}{6}\left(\frac{x-a_1}{a_2-a_1}\right) & ; \quad a_1 \le x \le a_2 \\ \frac{1}{6} + \frac{1}{6}\left(\frac{x-a_2}{a_3-a_2}\right) & ; \quad a_2 \le x \le a_3 \\ \frac{2}{6} + \frac{1}{6}\left(\frac{x-a_3}{a_4-a_3}\right) & ; \quad a_3 \le x \le a_4 \\ \frac{3}{6} + \frac{1}{6}\left(\frac{x-a_4}{a_5-a_4}\right) & ; \quad a_4 \le x \le a_5 \\ \frac{4}{6} + \frac{1}{6}\left(\frac{x-a_5}{a_6-a_5}\right) & ; \quad a_5 \le x \le a_6 \\ \frac{5}{6} + \frac{1}{6}\left(\frac{x-a_6}{a_7-a_6}\right) & ; \quad a_6 \le x \le a_7 \\ 1 - \frac{1}{6}\left(\frac{x-a_7}{a_8-a_7}\right) & ; \quad a_7 \le x \le a_8 \\ \frac{5}{6} - \frac{1}{6}\left(\frac{x-a_8}{a_9-a_8}\right) & ; \quad a_8 \le x \le a_9 \\ \frac{4}{6} - \frac{1}{6}\left(\frac{x-a_9}{a_{10}-a_9}\right) & ; \quad a_9 \le x \le a_{10} \\ \frac{3}{6} - \frac{1}{6}\left(\frac{x-a_{10}}{a_{11}-a_{10}}\right) & ; \quad a_{10} \le x \le a_{11} \\ \frac{2}{6} - \frac{1}{6}\left(\frac{x-a_{11}}{a_{12}-a_{11}}\right) & ; \quad a_{11} \le x \le a_{12} \\ \frac{1}{6}\left(\frac{a_{13}-x}{a_{13}-a}\right) & ; \quad a_{12} \le x \le a_{13} \\ 0 & ; \quad x > a_{13} \end{cases}$$

Parameteric Form Of Tridecagonal Fuzzy Number

The tridecagonal fuzzy number can also be stated in the parametric form as U = ($A_1(r)$, $B_1(s)$, $C_1(t)$, $D_1(u)$, $E_1(v)$, $F_1(w)$, $A_2(r)$, $B_2(s)$, $C_2(t)$, $D_2(u)$, $E_2(v)$, $F_2(w)$) for r ϵ [0,1/6], s ϵ [1/6,2/6], t ϵ [2/6,3/6], u ϵ [3/6,4/6], v ϵ [4/6,5/6], w ϵ [5/6,1] where,

(a) $A_1(r)$, $B_1(s)$, $C_1(t)$, $D_1(u)$, $E_1(v)$ and $F_1(w)$ are bounded left continuous non-decreasing function over [0,1/6], [1/6,2/6], [2/6,3/6], [3/6,4/6], [4/6,5/6],and [5/6,1].

(b) $A_2(r)$, $B_2(s)$, $C_2(t)$, $D_2(u)$, $E_2(v)$ and $F_2(w)$ are bounded left continuous non-increasing function over [0,1/6], [1/6,2/6], [2/6,3/6], [3/6,4/6], [4/6,5/6],and [5/6,1].

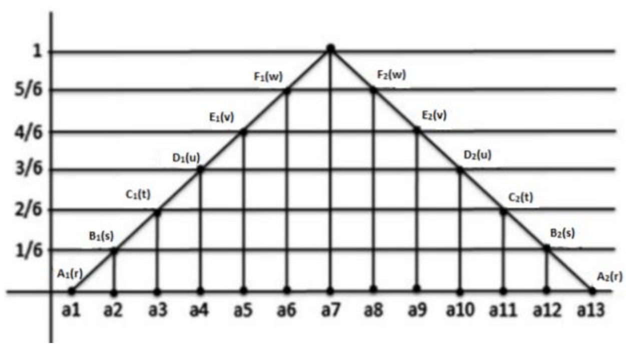

Figure 1: Graphical representation of Tridecagonal Fuzzy Number

Positive Tridecagonal Fuzzy Number

The positive tridecagonal fuzzy number $\bar{A}_{TD}$ is denoted as $\bar{A}_{TD}$= (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}) where all $a_i > 0$ for all i = 1,2,3,4,5,6,7,8,9,10,11,12,13. For example, $\bar{A}_{TD}$=(1,3,5,7,9,11,13,15,17,19,21,23,25).

Negative Tridecagonal Fuzzy Number

The negative tridecagonal fuzzy number $\bar{A}_{TD}$ is denoted as $\bar{A}_{TD}$= (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}) where all $a_i < 0$ for all i = 1,2,3,4,5,6,7,8,9,10,11,12,13. For example, $\bar{A}_{TD}$=(-1,-3,-5,-7,-9,-11,-13,-15,-17,-19,-21,-23,-25).

ARITHMETIC OPERATIONS ON TRIDECAGONAL FUZZY NUMBERS

Let us consider two tridecagonal fuzzy numbers $\bar{A}_{TD}$= (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}) and $\bar{B}_{TD}=(b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8,b_9,b_{10},b_{11},b_{12},b_{13})$.

Addition

$\bar{A}_{TD} + \bar{B}_{TD}=((a_1+b_1), (a_2+b_2), (a_3+b_3), (a_4+b_4), (a_5+b_5), (a_6+b_6), (a_7+b_7), (a_8+b_8), (a_9+b_9), (a_{10}+b_{10}), (a_{11}+b_{11}), (a_{12}+b_{12}), (a_{13}+b_{13}))$

Subtraction

$\bar{A}_{TD}-\bar{B}_{TD} =((a_1-b_{13}), (a_2-b_{12}), (a_3-b_{11}), (a_4-b_{10}), (a_5-b_9), (a_6-b_8), (a_7-b_7), (a_8-b_6), (a_9-b_5), (a_{10}-b_4), (a_{11}-b_3), (a_{12}-b_2), (a_{13}-b_1))$

Scalar Multiplication

$$\alpha\bar{A}_{TD}= \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha a_5, \alpha a_6, \alpha a_7, \alpha a_8, \alpha a_9, \alpha a_{10}, \alpha a_{11},\alpha a_{12}, \alpha a_{13}) \text{ if } \alpha> 0 \\ (\alpha a_{13}, \alpha a_{12}, \alpha a_{11}, \alpha a_{10}, \alpha a_9, \alpha a_8, \alpha a_7, \alpha a_6, \alpha a_5, \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1) \text{ if } \alpha< 0 \end{cases}$$

RANKING OF TRIDECAGONAL FUZZY NUMBERS

The mapping $r:F(R) \longrightarrow R$ where F(R) is a set of fuzzy numbers which is defined as a set of real numbers R, and maps every fuzzy number into the real line, without affecting the natural order is known as the ranking function, i.e.

(a) $\bar{A}_{TD} > \bar{B}_{TD}$ iff $r(\bar{A}_{TD}) > r(\bar{B}_{TD})$
(b) $\bar{A}_{TD} < \bar{B}_{TD}$ iff $r(\bar{A}_{TD}) < r(\bar{B}_{TD})$
(c) $\bar{A}_{TD} = \bar{B}_{TD}$ iff $r(\bar{A}_{TD}) = r(\bar{B}_{TD})$

Let us consider two tridecagonal fuzzy numbers $\bar{A}_{TD}$= (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}) and $\bar{B}_{TD}=(b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8,b_9,b_{10},b_{11},b_{12},b_{13})$ then

$$r(\bar{A}_{TD}) = \frac{a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}+a_{11}+a_{12}+a_{13}}{13}$$

and

$$r(\bar{B}_{TD}) = \frac{b_1+b_2+b_3+b_4+b_5+b_6+b_7+b_8+b_9+b_{10}+b_{11}+b_{12}+b_{13}}{13}$$

SOLUTION OF ALL 2x2 MATRIX GAME

Let us consider the general 2x2 game matrix. The payoffs used in the game matrix are represented as tridecagonal fuzzy number.

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Algorithm

For solving a Fuzzy Game Problem follow the steps given below,
Step 1: First of all convert the fuzzy game problem into a crisp value problem using ranking function.
Step 2: Test for saddle point in the fuzzy payoff matrix.
Step 3: If there is no saddle point in the payoff matrix then solve it by finding optimal strategies.

The optimal mixed strategies for player A = (p_1, p_2) and for player B = (q_1, q_2), where

$$p_1 = \frac{(a_{22} - a_{12})}{(a_{11} + a_{22}) - (a_{12} + a_{21})} \;;\; p_2 = 1 - p_1 \text{ and } q_1 = \frac{(a_{22} - a_{21})}{(a_{11} + a_{22}) - (a_{12} + a_{21})} \;;\; q_2 = 1 - q_1$$

And,

$$\text{Value of the game } (V) = \frac{(a_{11}a_{22} - a_{12}a_{21})}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

NUMERICAL EXAMPLES

Example 1:
Consider the following fuzzy game problem

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} (1,2,3,4,5,6,7,8,9,10,11,12,13) & (1,2,4,5,6,8,9,10,14,15,16,17) \\ (1,3,5,7,9,11,13,15,17,19,21,23,25) & (2,4,6,8,10,12,14,16,18,20,22,24,26) \end{pmatrix} \end{array}$$

Solution:
Step 1: By definition of tridecagonal fuzzy numbers $r(\bar{A}_{TD})$ is calculated as

$$r(\bar{A}_{TD}) = \frac{a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}+a_{11}+a_{12}+a_{13}}{13}$$

$a_{11} = (1,2,3,4,5,6,7,8,9,10,11,12,13)$	$r(a_{11}) = \frac{1+2+3+4+5+6+7+8+9+10+11+12+13}{13} = \frac{91}{13} = 7$
$a_{12} = (1,2,4,5,6,8,9,10,14,15,16,17)$	$r(a_{12}) = \frac{1+2+4+5+6+8+9+10+14+15+16+17}{13} = \frac{130}{13} = 10$
$a_{21} = (1,3,5,7,9,11,13,15,17,19,21,23,25)$	$r(a_{21}) = \frac{1+3+5+7+9+11+13+15+17+19+21+23+25}{13} = \frac{169}{13} = 13$
$a_{22} = (2,4,6,8,10,12,14,16,18,20,22,24,26)$	$r(a_{22}) = \frac{2+4+6+8+10+12+14+16+18+20+22+24+26}{13} = \frac{182}{13} = 14$

Step 2: The payoff matrix is

$$\begin{array}{ccc} & \text{Player B} & \text{Row Minimum} \\ \text{Player A} & \begin{pmatrix} 7 & 10 \\ 13 & 14 \end{pmatrix} & \begin{array}{c} 7 \\ 13 \end{array} \\ \text{Column Maximum} & \begin{array}{cc} 13 & 14 \end{array} & \end{array}$$

Min-Max = 13 and Max-Min = 13
Since, the Min-Max = Max-Min. It has a saddle point. The crisp solution to the problem is saddle point = (A_2,B_1). Therefore, value of the game = 13.

Example 2:

Consider the following fuzzy game problem

$$\text{Player A} \quad \begin{array}{c} \text{Player B} \\ \begin{pmatrix} (2,4,5,6,7,8,9,12,14,15,18,20,23) & (5,8,10,11,12,13,15,16,17,19,21,22,26) \\ (6,8,10,11,12,14,15,16,18,20,24,26,28) & (3,4,5,7,9,11,13,14,16,17,18,19,20) \end{pmatrix} \end{array}$$

Solution:

Step 1: By definition of tridecagonal fuzzy numbers $r(\bar{A}_{TD})$ is calculated as

$$r(\bar{A}_{TD}) = \frac{a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9+a_{10}+a_{11}+a_{12}+a_{13}}{13}$$

$a_{11} = (2,4,5,6,7,8,9,12,14,15,18,20,23)$	$r(a_{11}) = \frac{2+4+5+6+7+8+9+12+14+15+18+20+23}{13} = \frac{142}{13} = 11$
$a_{12} = (5,8,10,11,12,13,15,16,17,19,21,22,26)$	$r(a_{12}) = \frac{5+8+10+11+12+13+15+16+17+19+21+22+26}{13} = \frac{195}{13} = 15$
$a_{21} = (6,8,10,11,12,14,15,16,18,20,24,26,28)$	$r(a_{21}) = \frac{6+8+10+11+12+14+15+16+18+20+24+26+28}{13} = \frac{208}{13} = 16$
$a_{22} = (3,4,5,7,9,11,13,14,16,17,18,19,20)$	$r(a_{22}) = \frac{3+4+5+7+9+11+13+14+16+17+18+19+20}{13} = \frac{156}{13} = 12$

Step 2: The payoff matrix is

$$\begin{array}{lcc} & \text{Player B} & \text{Row Minimum} \\ \text{Player A} & \begin{pmatrix} 11 & 15 \\ 16 & 12 \end{pmatrix} & \begin{array}{c} 11 \\ 12 \end{array} \\ \text{Column Maximum} & 16 \quad 15 & \end{array}$$

Min-Max = 15 and Max-Min = 12

Since, Min-Max is not equal to Max-Min, hence there is no saddle point.

Step 3: Now we shall find the optimum mixed strategy and the value of the game,

Here, $a_{11} = 11$, $a_{12} = 15$, $a_{21} = 16$, $a_{22} = 12$.

$$p_1 = \frac{(a_{22} - a_{12})}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{12 - 15}{(11 + 12) - (15 + 16)} = \frac{3}{8} \quad \text{and} \quad p_2 = 1 - p_1 = 1 - \frac{3}{8} = \frac{5}{8}$$

$$q_1 = \frac{(a_{22} - a_{21})}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{12 - 16}{(11 + 12) - (15 + 16)} = \frac{1}{2} \quad \text{and } q_2 = 1 - q_1 = 1 - \frac{1}{2} = \frac{1}{2}$$

Therefore, Optimal mixed strategies are

Player A = $(p_1, p_2) = (3/8, 5/8)$ and Player B = $(q_1, q_2) = (1/2, 1/2)$.

$$\text{Value of the game (V)} = \frac{(a_{11}a_{22} - a_{12}a_{21})}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{11\text{x}12 - 15\text{x}16}{(11 + 12) - (15 + 16)} = \frac{108}{8} = 13.5$$

CONCLUSION

In this paper, the method for solving a fuzzy game problem using ranking of tridecagonal fuzzy number has been adopted. The optimal solution of the fuzzy game is obtained by using standard method. In the above two examples we have considered only (2x2) fuzzy game but the method applied here can also be used to solve any (mxn) fuzzy game problem. In future, this type of fuzzy game problem can be solved using another fuzzy number.

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