ON SOME NOVEL CORRELATION COEFFICIENTS IN PICTURE FUZZY ENVIRONMENT

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Abstract

Picture fuzzy set (PFS) is an important tool for handling uncertainty and vagueness, particularly in situations that require more answers of the type “yes”,” no”, “abstain”, “refusal”. Correlation coefficient of PFSs is an essential measure in picture fuzzy set theory and has a lot of applications in many areas, such as “decision-making”, “medical diagnosis”, “pattern recognition” etc. In the existing studies related to correlation coefficients of PFSs, the value of correlation coefficients is one even if the two PFSs are not equal. Also, in some problems, the value comes out to be indeterminate. In this article, two new correlation coefficients of PFSs are introduced along with some of their properties. These correlation coefficients of PFSs are not only better than existing ones but also effective in dealing with some practical problems where existing correlation coefficients fail.

Keywords: Atannassov’s intuitionistic fuzzy set, correlation coefficient, picture fuzzy set, pattern recognition

1. INTRODUCTION

To describe incomplete, uncertain or inaccurate information, fuzzy sets introduced by Zadeh (1965) play a key role. But, Zadeh’s fuzzy sets are not competent when there is a deficiency of knowledge of the degrees of membership. So, Atanassov (1986) generalized the Zadeh’s fuzzy sets and introduced the concept of intuitionistic fuzzy sets (IFSs) which considers membership as well as non-membership of an element together with their sum being less or equal to one. In the last two decades, lots of developments regarding IFSs came up (Szmidt & Kacprzyk, 2002; Xu, 2007; Wei, 2015; Ji et al., 2018; Liu & Zhang, 2018). But, the applications of IFSs are limited due to the fact that they are capable of only handling the vague concepts such as “neither this nor that”. So, based on this reason, Cuong(2013) introduced the concept of picture fuzzy sets (PFSs) which is a direct generalization of Zadeh’s fuzzy sets and Atanassov’s IFSs. A picture fuzzy set (PFS) is characterized by the degree of membership, degree of non-membership and degree of neutrality with the condition that the sum of these degrees should be less or equal to one. PFS can precisely explicit the opinions of decision-makers’ including no, yes, abstain, and refusal. Therefore, avoids the missing valuation information and enhancing the uniformity of the acquired information with real decision-making environment.

Presently, the research about PFSs including their extensions mostly focus on aggregation operators and information measures along with their application to MCDM problems and clustering analysis (Singh, 2015; Thong, 2016; Wei et al., 2016; Wei, 2016, 2017; Son, 2017; Wang et al., 2017; Peng, 2017; Wang et al., 2018a, 2018b; Zhang et al., 2019; Yang et al., 2019; Wang et al., 2019a). The present study is devoted to the formulation of effective correlation coefficients in picture fuzzy theory.

In the intuitionistic fuzzy theory, correlation measurement of two IFSs plays a key role and it is usually expressed in terms of a correlation coefficient. Many researchers have given due attention to the investigation of the intuitionistic fuzzy correlation coefficient and successfully achieved many valuable results. For example, Hung (2001) introduced the coefficient of correlation for IFSs from the viewpoint of statistics. Hung and Wu (2002) developed a method for calculating the coefficient of correlation for IFSs by means of “centroid” method, which not only reflected their degree of correlation but also their nature (positive or negative) of correlation. Furthermore, they also introduced the “centroid” method for interval-valued intuitionistic fuzzy sets (IVIFSs). Xu (2006) proposed a coefficient of correlation for IFSs and also extended it to IVIFSs. An intuitionistic fuzzy correlation coefficient taking all the three dimensions i.e., membership, non-membership, and hesitation into consideration was also introduced by Xu (2012). In picture fuzzy setting, Singh (2015) introduced two correlation coefficients and applied them in picture fuzzy clustering analysis and bidirectional approximate reasoning.

Most of the correlation coefficients in fuzzy/non-standard fuzzy settings often give one as index of correlation even if the two sets are not equal. Also, the correlation coefficients introduced by Xu (2006, 2012) reduce to $0/0$ form when the two sets are same, which is not meaningful in mathematical logic. So, these factors motivated us to introduce some new effective picture fuzzy correlation coefficients. Therefore, in this study, we propose two new picture fuzzy correlation coefficients together with some of their properties. Furthermore, we apply the proposed picture fuzzy correlation coefficients in pattern recognition and also established their superiority over some existing picture fuzzy correlation coefficients with the help of illustrative examples. The main contribution of this study is:

- We introduce two new and effective picture fuzzy correlation coefficients along with some of their properties and advantages.
- We show the application of the proposed picture fuzzy correlation coefficients in pattern recognition.
- We establish the superiority of our proposed picture fuzzy correlation coefficients by considering some comparative studies.

The remainder of this paper is organized as follows:

Section 2 recalls some basic concepts related to fuzzy/non-standard fuzzy theory. Two new picture fuzzy correlation coefficients and some of their properties are introduced in section 3. Section 4 shows the application and superiority of the proposed picture fuzzy correlation coefficients. Finally, the paper is concluded in the section 5.

2. PRELIMINARIES

In this section, we recall some definitions and formulae that have been used in this paper.

Definition 1 (Atanassov, 1986) Let $U = \{u_1, u_2, \dots, u_t\}$ be a finite universe of discourse, then

$P = \{(u_i, \gamma_P(u_i), \delta_P(u_i)) | u_i \in U, i = 1, 2, \dots, t\}$is known as Atanassovs' IFS where $\gamma_P(u_i)$ and $\delta_P(u_i)$ represent the membership and non-membership degrees respectively of the element $u_i \in U$ to set P with the conditions

$0 \le \gamma_P(u_i) \le 1, 0 \le \delta_P(u_i) \le 1$and $0 \le \gamma_P(u_i) + \delta_P(u_i) \le 1$.

Definition 2(Cuong, 2013) A picture fuzzy set P on a finite universe of discourse $U = \{u_1, u_2, \dots, u_t\}$ is defined as$P = \{(u_i, \gamma_P(u_i), \theta_P(u_i), \delta_P(u_i)) | u_i \in U, i = 1, 2, \dots, t\}$,

where,$\gamma_P(u_i), \theta_P(u_i)$and $\delta_P(u_i)$ denote the degree of membership, degree of neutrality and degree of non-membership of the element $u_i \in U$ to set P respectively with the condition $0 \le \gamma_P(u_i) + \theta_P(u_i) + \delta_P(u_i) \le 1$.

Definition 3 (Singh, 2015) Let

$P = \{(u_i, \gamma_P(u_i), \theta_P(u_i), \delta_P(u_i)) | u_i \in U, i = 1, 2, \dots, t\}$and

$Q = \left\{\left(u_i, \gamma_Q(u_i), \theta_Q(u_i), \delta_Q(u_i)\right) | u_i \in U, i = 1, 2, \dots, t\right\}$be two PFSs on the universe of discourse $U = \{u_1, u_2, \dots, u_t\}$, then the correlation coefficient between P and Qis

$$K(P, Q) = \frac{\sum_{i=1}^{t}\{\gamma_P(u_i)\gamma_Q(u_i) + \theta_P(u_i)\theta_Q(u_i) + \delta_P(u_i)\delta_Q(u_i) + \rho_P(u_i)\rho_Q(u_i)\}}{\left\{\left\{\sum_{i=1}^{t}\left((\gamma_P(u_i))^2 + (\theta_P(u_i))^2 + (\delta_P(u_i))^2 + (\rho_P(u_i))^2\right)\right\}^{\frac{1}{2}} \times \left\{\sum_{i=1}^{t}\left(\left(\gamma_Q(u_i)\right)^2 + \left(\theta_Q(u_i)\right)^2 + \left(\delta_Q(u_i)\right)^2 + (\rho_Q(u_i))^2\right)\right\}^{\frac{1}{2}}\right\}} \quad (1)$$

where$\rho_P(u_i) = 1 - (\gamma_P(u_i) + \theta_P(u_i) + \delta_P(u_i))$ for all $i = 1, 2, \dots, t$ is the degree of refusal.

Another correlation coefficient between the above mentioned PFSs P and Q is

$$\mathrm{K}_1(\mathrm{P}, Q) = \frac{\sum_{i=1}^{t}\{\gamma_P(u_i)\gamma_Q(u_i) + \theta_P(u_i)\theta_Q(u_i) + \delta_P(u_i)\delta_Q(u_i) + \rho_P(u_i)\rho_Q(u_i)\}}{max\left\{\sum_{i=1}^{t}\left((\gamma_P(u_i))^2 + (\theta_P(u_i))^2 + (\delta_P(u_i))^2 + (\rho_P(u_i))^2\right), \sum_{i=1}^{t}\left(\left(\gamma_Q(u_i)\right)^2 + \left(\theta_Q(u_i)\right)^2 + \left(\delta_Q(u_i)\right)^2 + (\rho_Q(u_i))^2\right)\right\}} \quad (2)$$

In the next section, we introduce two novel picture fuzzy correlation coefficients.

3. NEW CORRELATION COEFFICIENTS FOR PFSS

In this section, we introduce two new and effective picture fuzzy correlation coefficients.

Let $P_1 = \left\{\left(u_i, \gamma_{P_1}(u_i), \theta_{P_1}(u_i), \delta_{P_1}(u_i)\right) / u_i \in U\right\}$ and$P_2 = \left\{\left(u_i, \gamma_{P_2}(u_i), \theta_{P_2}(u_i), \delta_{P_2}(u_i)\right) / u_i \in U\right\}$ be two PFSs in the universe of discourse$U = \{u_1, u_2, \dots, u_t\}$. We define the correlation coefficient between the two PFSs P_1 and P_2 as

$\psi(P_1, P_2) = \frac{1}{3t}\sum_{i=1}^{t}[\lambda_i(1 - \Delta\gamma_i) + \mu_i(1 - \Delta\theta_i) + \nu_i(1 - \Delta\delta_i)]$(3)

where

$\lambda_i = \frac{c - \Delta\gamma_i - \Delta\gamma_{max}}{c - \Delta\gamma_{min} - \Delta\gamma_{max}}$, $\mu_i = \frac{c - \Delta\theta_i - \Delta\theta_{max}}{c - \Delta\theta_{min} - \Delta\theta_{max}}$, $\vartheta_i = \frac{c - \Delta\delta_i - \Delta\delta_{max}}{c - \Delta\delta_{min} - \Delta\delta_{max}} (c > 2, i = 1, 2, \dots, t)$,

$\Delta\gamma_i = \left|\gamma_{P_1}(u_i) - \gamma_{P_2}(u_i)\right|$, $\Delta\theta_i = \left|\theta_{P_1}(u_i) - \theta_{P_2}(u_i)\right|$, $\Delta\delta_i = \left|\delta_{P_1}(u_i) - \delta_{P_2}(u_i)\right|$,

$\Delta\gamma_{min} = \min_i\left\{\left|\gamma_{P_1}(u_i) - \gamma_{P_2}(u_i)\right|\right\}$, $\Delta\gamma_{max} = \max_i\left\{\left|\gamma_{P_1}(u_i) - \gamma_{P_2}(u_i)\right|\right\}$,

$\Delta\delta_{min} = \min_i\left\{\left|\delta_{P_1}(u_i) - \delta_{P_2}(u_i)\right|\right\}$, $\Delta\delta_{max} = \max_i\left\{\left|\delta_{P_1}(u_i) - \delta_{P_2}(u_i)\right|\right\}$,

$\Delta\theta_{max} = \max_i\left\{\left|\theta_{P_1}(u_i) - \theta_{P_2}(u_i)\right|\right\}$,$\Delta\theta_{min} = \min_i\left\{\left|\theta_{P_1}(u_i) - \theta_{P_2}(u_i)\right|\right\}$.

By considering the degree of refusal $\rho_P(u_i) = 1 - \left(\gamma_P(u_i) + \theta_P(u_i) + \delta_P(u_i)\right)$, another correlation coefficient for the above two PFSs P_1 and P_2 is defined as

$$\psi_1(P_1, P_2) = \frac{1}{4t}\sum_{i=1}^{t}[\lambda_i(1 - \Delta\gamma_i) + \mu_i(1 - \Delta\theta_i) + \nu_i(1 - \Delta\delta_i) + \xi_i(1 - \Delta\rho_i)] \quad (4)$$

where, $\xi_i = \frac{c - \Delta\rho_i - \Delta\rho_{max}}{c - \Delta\rho_{min} - \Delta\rho_{max}}$,$\Delta\rho_i = \left|\rho_{P_1}(u_i) - \rho_{P_2}(u_i)\right|$, $\Delta\rho_{min} = \min_i\left\{\left|\rho_{P_1}(u_i) - \rho_{P_2}(u_i)\right|\right\}$,

$\Delta\rho_{max} = \max_i\left\{\left|\rho_{P_1}(u_i) - \rho_{P_2}(u_i)\right|\right\}$, and the rest of the terms are same as in correlation coefficient (3).

Remark 1 The condition "$c > 2$" guarantees that $0 < \lambda_i, \mu_i, \nu_i, \xi_i < 1$. Therefore, the coefficients of correlation $\psi(P_1, P_2)$and$\psi_1(P_1, P_2)$ satisfy$0 \le \psi(P_1, P_2), \psi_1(P_1, P_2) \le 1$.

Now, we discuss some properties of the proposed correlation coefficient$\psi(P_1, P_2)$ in the following.

Theorem 1: *The correlation coefficient $\psi(P_1, P_2)$ satisfies the following properties:*

(a) $0 \le \psi(P_1, P_2) \le 1$;
(b) $\psi(P_1, P_2) = \psi(P_2, P_1)$;
(c) $\psi(P_1, P_2) = 1$ *if and only if* $P_1 = P_2$.

Proof(a) Since $0 \le \lambda_i \le 1$, $0 \le \mu_i \le 1$, $0 \le \nu_i \le 1$ and $0 \le 1 - \Delta\gamma_i \le 1$, $0 \le 1 - \Delta\theta_i \le 1$,

$0 \le 1 - \Delta\delta_i \le 1$, so $0 \le \lambda_i(1 - \Delta\gamma_i) + \mu_i(1 - \Delta\theta_i) + \nu_i(1 - \Delta\delta_i) \le 3 (i = 1, 2, \dots, t)$.

Therefore by (3), we have $0 \le \psi(P_1, P_2) \le 1$.

(b) It is straight forward.

(c) Suppose that $P_1 = P_2$, then $\gamma_{P_1}(u_i) = \gamma_{P_2}(u_i)$, $\theta_{P_1}(u_i) = \theta_{P_2}(u_i)$, $\delta_{P_1}(u_i) - \delta_{P_2}(u_i)$ for all

$u_i \in U$ $(i = 1, 2, \dots, t)$and thus

$\Delta\gamma_i = \Delta\gamma_{min} = \Delta\gamma_{max} = 0$, $\Delta\theta_i = \Delta\theta_{min} = \Delta\theta_{max} = 0$, , $\Delta\delta_i = \Delta\delta_{min} = \Delta\delta_{max} = 0$.

Therefore by (3), $\psi(P_1, P_2) = 1$.

Conversely, suppose that$\psi(P_1, P_2) = 1$.

As$0 \le \lambda_i(1 - \Delta\gamma_i) \le 1, 0 \le \mu_i(1 - \Delta\theta_i) \le 1, 0 \le \nu_i(1 - \Delta\delta_i) \le 1$,

so, we have

$\lambda_i(1-\Delta\gamma_i)+\mu_i(1-\Delta\theta_i)+\nu_i(1-\Delta\delta_i)=3$and $\lambda_i(1-\Delta\gamma_i)=\mu_i(1-\Delta\theta_i)=(1-\Delta\delta_i)=1(i=1,2,\dots,t)$.

Also, $0<\lambda_i\le 1, 0\le\mu_i\le 1, 0\le\nu_i\le 1, 0\le(1-\Delta\gamma_i)\le 1, 0\le(1-\Delta\theta_i)\le 1, 0\le(1-\Delta\delta_i)\le 1$

Therefore, we obtain

$\lambda_i=\mu_i=\nu_i=1, 1-\Delta\gamma_i=1-\Delta\theta_i=1-\Delta\delta_i=1$, i.e., $\Delta\gamma_i=\Delta\theta_i=\Delta\delta_i=0$.

Hence, $\gamma_{P_1}(u_i)=\gamma_{P_2}(u_i)$, $\theta_{P_1}(u_i)=\theta_{P_2}(u_i)$, $\delta_{P_1}(u_i)-\delta_{P_2}(u_i)$ for all $u_i\in U$, i.e., $P_1=P_2$.

The correlation coefficient (4) has the same properties as that of the correlation coefficient (3).

Remark 2 The properties if correlation coefficient $\psi_{\mathbf{1}}(P_1,P_2)$ can be proved in the similar manner.

In the next section, we discuss the application and superiority of our proposed picture fuzzy correlation coefficient$\psi(P_1,P_2)$.

4. APPLICATION

In this section, we show the application of the proposed correlation coefficients for PFSs in pattern recognition and also compare the results with some existing correlation measures in picture fuzzy environment.

4.1 Pattern Recognition

In pattern recognition, an unknown pattern is classified into some given known patterns. For this purpose, various fuzzy information measures like fuzzy similarity measure, fuzzy distance measure, fuzzy divergence measure, etc. are utilized. Here we use our proposed correlation measures for classifying an unknown pattern to one of the given known patterns. We also use correlation coefficients given by other researchers for establishing the superiority of our proposed correlation coefficients.

In general, we can formulate a pattern recognition problem in the picture fuzzy environment as follows.

Problem formulation: Suppose $\{P_1,P_2,\dots,P_t\}$ be some known patterns characterized by picture fuzzy sets in the universal set $U=\{u_1,u_2,\dots,u_t\}$as follows:

$$P_i=\left\{\left(u_i,\gamma_{P_1}(u_i),\theta_{P_1}(u_i),\delta_{P_1}(u_i)\right)|u_i\in U, i=1,2,\dots,t\right\}$$

Let $R=\{(u_i,\gamma_R(u_i),\theta_R(u_i),\delta_R(u_i))|u_i\in U, i=1,2,\dots,t\}$be an unknown pattern. The problem is to classify the pattern R to one of the known patterns P_i $(i=1,2,\dots,t)$.The solution to the problem can be obtained as follows:

Recognition principle: Let $C(P_i,R)$ be the picture fuzzy correlation coefficient of $R$from$P_i(i=1,2,\dots,t)$. Then R is assigned to $P_i{}^*$ where

$i^*=\arg\max_i\{C(P_i,R)\}, i=1,2,\dots,t.$

Now, we investigate the performance of our proposed picture fuzzy correlation coefficients with the help of the illustrative examples.

4.1.1 Comparison of our proposed correlation coefficients (3) and (4)with Singh's correlation coefficient (1)

Example 1 Consider two known patterns P_1 and P_2 that are given in terms of picture fuzzy setsin a universe of discourse $U=\{u_1,u_2,u_3\}$ as

$P_1=\{(u_1,0.28,0.32,0.24),(u_2,0.11,0.29,0.34),(u_3,0.87,0.12,0.01)\}$

$P_2=\{(u_2,0.25,0.24,0.13),(u_2,0.41,0.21,0.23),(u_3,0.32,0.23,0.24)\}$

Let Q be an unknown pattern given in terms of picture fuzzy set as

$Q=\{(u_1,0.12,0.25,0.24),(u_2,0.22,0.25,0.24),(u_3,0.54,0.32,0.11)\}$

The problem is to classify the unknown pattern Q to one of the known patterns P_1or P_2. For this purpose, we use our proposed correlation coefficients (3) and (4) and Singh's correlation coefficient (1) for picture fuzzy sets. We have $K(P_1, Q) = K(P_2, Q) = 0.8935; \psi(P_1, Q) = 0.8553, \psi(P_2, Q) = 0.8807$ and $\psi_1(P_1, Q) = 0.8626$, $\psi_1(P_2, Q) = 0.8756$ (Here *c*= 3). It is clear that Singh's correlation coefficient (1) cannot be applied for classifying the unknown pattern Qinto one of the known patterns P_1or P_2whereas our proposed correlation coefficients (3) and (4) classifies $Q$to P_2.

4.1.2 Comparison of our proposed correlation coefficients (3) and (4) with Singh's correlation coefficient (2)

Example 2 Let P_1 and P_2 be two known patterns in a finite universe of discourse $U = \{u_1, u_2, u_3\}$ given in the form of picture fuzzy sets as

$P_1 = \{(u_1, 0.17, 0.22, 0.24), (u_2, 0.27, 0.15, 0.32), (u_3, 0.80, 0.12, 0.00)\}\ and$

$P_2 = \{(u_2, 0.19, 0.29, 0.11), (u_2, 0.41, 0.23, 0.24), (u_3, 0.32, 0.23, 0.24)\}$

Consider an unknown pattern Q given in terms of picture fuzzy set as

$Q = \{(u_1, 0.40, 0.23, 0.00), (u_2, 0.22, 0.30, 0.27), (u_3, 0.54, 0.32, 0.11)\}$.

By utilizing Singh's correlation coefficient (2) for picture fuzzy sets and our proposed correlations (3) and (4), we calculate the correlation between the unknown pattern Q and the known patterns P_1 and P_2.We have $K_1(P_1, Q) = K_1(P_2, Q) = 0.8128$; $\psi(P_1, Q) = 0.8245, \psi(P_2, Q) = 0.8645$ and $\psi_1(P_1, Q) = 0.8574$, $\psi_2(P_2, Q) = 0.8677$ (Here *c*= 3). It is clear that Singh's correlation coefficient (2) cannot be applied for classifying the unknown pattern Qinto one of the known patterns P_1or P_2whereas our proposed correlation coefficients (3) and (4) classifies $Q$to P_2.

5. CONCLUSION

In this article, we proposed two new and effective picture fuzzy correlation coefficients. We have discussed some of their properties as well. We found that the value of our proposed correlation coefficients is one if and only if the two PFSs are equal, hence seems to be stronger correlation coefficients. Also, we applied our proposed picture fuzzy correlation coefficients in pattern recognition and compared the results with some existing picture fuzzy correlation coefficients. We have observed that the results obtained by our proposed picture fuzzy correlation coefficients are better than the results obtained by using some existing picture fuzzy correlation coefficients. Our future studies include 1) the generalization of picture fuzzy correlation coefficients 2) the development of correlation coefficients for picture fuzzy soft sets and hesitant picture fuzzy sets with their applications.

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