SELECTION OF STOCK PORTFOLIO USING INTEGRATED APPROACH OF AHP AND TOPSIS METHODS

Dinesh K. Sharma, University of Maryland Eastern Shore, USA
H.S. Hota, Guru Ghasidas Central University, India
Hari Sharma, Virginia State University, USA

ABSTRACT

Multi criteria decision making (MCDM) methods have been applied successfully in many domains including business and finance for obtaining optimal ranking based on preferred criteria, which are often conflicting. Analytical hierarchy process (AHP) and technique for order preference by similarity to ideal solution (TOPSIS) are two popular methods of MCDM that are useful for ranking and making selections where financial criteria are often conflicting. In this paper, we present an integrated approach of these two methods to select stock portfolio using a sample data with six criteria for six stocks. In this integrated approach, AHP is used to obtained weights and these weights are used in TOPSIS to evaluate ranking of stocks. Simulation software was developed to perform mathematical calculations. The integration of AHP and TOPSIS provided satisfactory results in ranking the stocks.

INTRODUCTION

Selection of stock portfolio is a complex decision making process, which requires knowledge and expertise. A successful investor or expert can decide a suitable set of stock to construct a portfolio that can produce the highest return for a given level of risk. Diversifying the investor's budget among the selected stocks is how a portfolio is developed. A portfolio can be constructed based on many fundamental and technical indicators associated with it. Since the fundamental and technical indicators are conflicting in nature, even an expert person can't easily select a set of stock that promises the maximum return. Although, Markowitz proposed a framework for mean-variance portfolio optimization in 1952, the researchers are always investigating to enhance the framework by applying sophisticated quantitative or qualitative techniques. Portfolio selection problem is considered as a multi criteria decision making (MCDM) problem in which the decision maker needs to select or rank available alternatives (stock) based on the conflicting nature of criteria (attribute) to find optimal solution. Analytical Hierarchy Process (AHP) (Saaty, 1980) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang and Yoon, 1981) are two very popular MCDM methods widely used in many decision making problems.

The review of literature reveals that numerous researchers have appealed MCDM methods in business and industry due to its theoretical development and practical applications (Hwang and Masud, 1979; Steuer, 1986; Sharma and Sharma, 2005). Many researchers have utilized MCDM methods in financial domain (Ehrgott et al., 2004; Lee et al., 2009). Ehrgott et al. (2004) have proposed a model for portfolio optimization using many technical indicators suggested by financial experts, based on multi-attribute utility theory and the classical mean-variance model for Markowitz. Kiris and Ustun (2010) have applied several fuzzy MCDM methods for portfolio selection problems in crisis environment using nine stocks of ISE.

In this paper, we combined original AHP with TOPSIS for construction of stock portfolio with a sample of six stocks with six attributes as criteria. AHP is firstly applied to find out weights after assigning weights based on Saaty's nine point scale by experts to form a relative importance matrix. These weights are then used to calculate final weights using TOPSIS method. A final ranking was obtained and verified from financial experts and found to be satisfactory.

INTEGRATION OF AHP AND TOPSIS

AHP Method: The main procedure of AHP using the radical root method (also called the geometric mean method) is as follows (Rao, 2007):
Step 1: Determine the objective and the evaluation attributes.

Step 2: Construct a pair-wise comparison matrix using a scale of relative importance. The judgments are entered using the fundamental scale of the analytic hierarchy process. An attribute compared with it is always assigned the value "1", so the main diagonal entries of the pair-wise comparison matrix are all "1" and the rating is based on Saaty's nine point scale shown in Table 1.

Table 1: Saaty's Nine Point Scale

Compare factor of i and j	Numerical rating
Extremely preferred	9
Very strongly preferred	7
Strongly preferred	5
Moderately preferred	3
Intermediate judgment between two adjacent judgments	2, 4, 6, 8

Assuming M attributes, the pair–wise comparison of attribute *i* with attribute *j* yields a square matrix $B_{M\times M}$ where a_{ij} denotes the comparative importance of attribute *i* with respect to attribute *j*. In the matrix b_{ij}=1 when *i*=*j* and $b_{ji}=\frac{1}{b_{ij}}$.

Find the relative normalized weight (W_j) of each attribute by
(i) Calculating the geometric mean of the i^{th} row, and
(ii) Normalizing the geometric means of rows in the comparison matrix. This can be represented as:

$$GM_j = \left[\prod_{i=1}^{M} b_{ij}\right]^{1/M} \quad (1)$$

$$\text{and } W_j = GM_j \Big/ \sum_{i=1}^{M} GM_j$$

Calculate matrices A3 and A4 such that A3=A1×A2 and A4= A3/A2, where A2=$[w_1,w_2,\ldots\ldots,w_i]^T$.
Determine the maximum Eigen value λ_{max} that is the average of matrix D4. Calculate the consistency index

$$CI= (\lambda_{max} - M) \Big/ (M-1) \quad (2)$$

Obtain the random index (RI) for the number of attributes used in decision making. Calculate the consistency ratio

$$CR = CI/RI \quad (3)$$

Step 3: The next step is to compare the alternatives pair-wise with respect to how much better they are in satisfying each of the attributes, i.e., to ascertain how well each alternative serves each attribute.

Step 4: In this step, we need to obtain the overall or composite performance scores for the alternatives by multiplying the relative normalized weight (W_j) of each attribute (obtained in step two) with its corresponding normalized weight value for each alternative (obtained in step three) and summing over the attributes for each alternative.

TOPSIS Method: Once the weights obtained through AHP are found consistence, we can use theses weights (Wj) as input to the TOPSIS which consists of the following steps (Rao, 2007):
Step 1: Obtain the normalized decision matrix,
Step 2: Obtain weighted normalized decision matrix,
Step 3: Obtain positive ideal solutions (PIS) and negative ideal solutions (NIS) as follows
Positive ideal solution:
$A^* = \{v_1^*, \ldots\ldots, v_n^*\}$,where

$$v_j^* = \left\{\max_i(v_{ij}) \text{ if } j \in J; \min_i(v_{ij}) \text{ if } j \in J'\right\} \quad (4)$$

Negative ideal solution :-
$A' = \{v_1', \ldots\ldots, v_n'\}$,where

$$v_j' = \left\{\min_i(v_{ij}) \text{ if } j \in J; \max_i(v_{ij}) \text{ if } j \in J'\right\} \quad (5)$$

Step 4: Obtain distance of each alternative (Separation measures) from PIS and NIS are calculated as follows:
The distance from the PIS:

$$S_i^* = \left[\sum_j \left(v_j^* - v_{ij}\right)^2\right]^{\frac{1}{2}}; i = 1, \ldots, m \quad (6)$$

Similarly, distance from NIS:

$$S_i' = \left[\sum_j \left(v_j' - v_{ij}\right)^2\right]^{\frac{1}{2}}; i = 1, \dots, m \tag{7}$$

Step 5: Obtain relative closeness Ci* to the ideal solution and finding rank of the alternatives based on value of Ci*.

$$C_i^* = \frac{S_i'}{S_i^* + S_i'} \tag{8}$$

EVALUATION OF STOCK PORTFOLIO

Overall process of stock portfolio selection and rank evaluation can be described using sub sections: (i) Identification of alternatives and criteria, (ii) Formation of decision hierarchy, (iii) Calculation of weight using AHP, and (iv) Evaluation of final ranking through TOPSIS.

(i) Identification of Alternatives and Criteria: Applying any of the MCDM methods in any domain requires an alternative as well as criteria. Alternatives in our case are the stocks in which the decision maker or investor wants to invest for high return; an investor wants to select a set of stocks to distribute the fund in such a manner so that a high return can be achieved. A set of six stocks is considered here as an alternatives for demonstration purposes. On the other hand, the selection of these stocks is based on some features known as criteria (Attribute). Many technical indicators are suggested by many financial experts; these are beta, P/E ratio, dividend, 1-year return, 3-year return and 5-year return. Details of criteria and its meaning are explained in Table 2. A sample data with 6 stocks (CVX, DIS, HPQ, IBM, INTC, JNJ) as shown in Table 3 are considered for stock portfolio selection.

Table 2 : Identified Criteria and Its Meaning

ID	Criteria	Meaning
C1	Beta	Beta is called systematic risk and is measured as the sensitivity of a security's returns to market returns.
C2	Dividend	Amount that a stock holder receives against each share
C3	P/E ratio	Shows relationship between a stock price and its company's earning
C4	One-year return	Percentage return in one year
C5	Three-year return	Percentage return in three years
C6	Five-year return	Percentage return in five years

Table 3: Normalized Objective Data for Stock Portfolio Selection with Six stocks (Alternatives) and Six Attributes (Criteria)

Stock	Beta	Dividend	P/E Ratio	1-Year Return (%)	3-Year Return (%)	5-Year Return (%)
CVX	0.53	1.00	0.50	0.31	0.25	0.52
DIS	0.97	0.13	0.69	1.00	0.11	0.32
HPQ	0.86	0.12	0.55	0.79	0.90	1.00
IBM	0.64	0.81	0.44	0.43	1.00	0.57
INTC	1.00	0.23	1.00	0.74	0.41	0.06
JNJ	0.48	0.72	0.51	0.46	0.36	0.07

(ii) Formation of Decision Hierarchy: A decision hierarchy as shown in Figure 1 is formed based on identified criteria and alternatives with objective as a root of the decision hierarchy, the next level of hierarchy consists of six criteria while leaves of hierarchy represents six alternatives (Stock) presented in Table 3. A decision hierarchy may have many levels with sub criteria. Let us represent criteria as C1, C2, C3, C4, C5, C6 and alternatives as At1, At2, At3, At4, At5 and At6.

(iii) Calculations of Weight Using AHP: Various steps of AHP method as explained above can be applied as described below:

Step 1: Objective data of attributes to be used as input data for AHP method are obtained from the authentic source. Out of 30 stocks with six attributes, only six stocks with all positive values are considered for demonstration purposes. Each attribute is normalized, so that the value of attribute will be in between 0 and 1 as shown in Table 3.

Step 2: A relative importance matrix as shown in Table 4 is constructed using Saaty's 9 point scale from Table 1 and based on the experience of financial experts. A financial expert assigns value of each attribute (A_{ij}) as per the requirement. Say for example an investor wants to select stock based on higher value of P/E ratio and dividend; however, another investor may assign a different value as per his/her own requirement for stock portfolio selection.

In the Table 4, Beta is strongly more important than P/E ratio in portfolio selection problem, so a relative importance value of 5 ($A_{12}=5$) is assigned to Beta (C1) over P/E ratio (C2) and a relative importance value 1/5=0.2 ($A_{21}=1/5$) is assigned to P/E (C2) ratio over Beta (C1). Similarly other values in the matrix are assigned based on the expert judgment of financial expert .In the matrix $A_{ij}=1$ for i=j, means when a criteria is compared with itself, relative importance value will be always 1. A different value may be assigned by a different expert depending upon the requirement. Also, geometric mean, consistency index (CI) and consistency ratio (CR) are calculated using equations 1, 2 and 3 respectively from the pair-wise comparison matrix and presented in Table 4. Calculated value of CR is 0.09, since the value of CR is less than 0.1, hence the weights assigned by the expert are consistent and can be used in the selection process to obtain final rank of stocks.

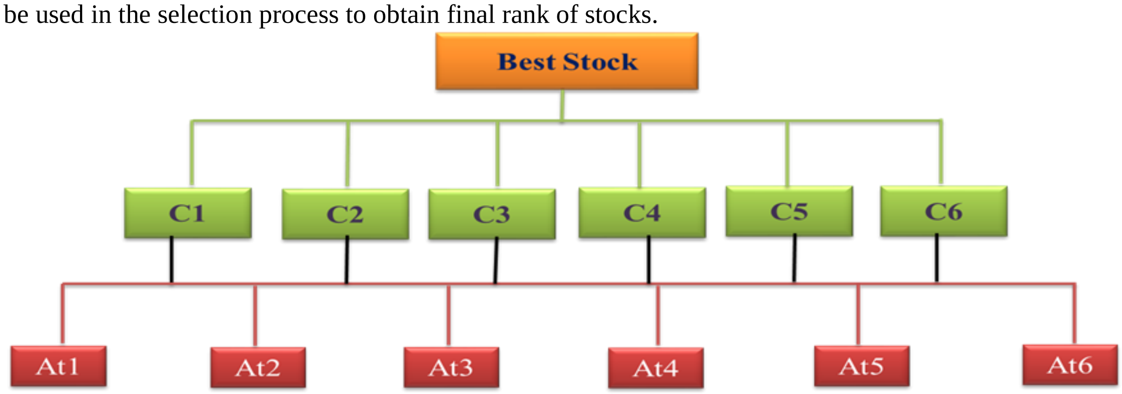

Figure 1: The Decision Hierarchy for Stock Portfolio Selection

Table 4: Relative Importance Matrix (Pair-wise Comparison)

A1	C1	C2	C3	C4	C5	C6	GM	Relative Normalized Weight (W=A2)	A3= A1*A2	A4= A3/A2	λ_{max}	CI	CR
C1	1	5	3	5	3	5	3.22	0.42	2.70	6.42	6.62	0.12	0.09
C2	0.2	1	0.33	1	1	1	0.63	0.08	0.51	6.18			
C3	0.33	3	1	3	3	3	1.73	0.22	1.42	6.32			
C4	0.2	1	0.33	1	5	3	1	0.13	0.96	7.36			
C5	0.33	1	0.33	0.2	1	3	0.63	0.08	0.58	6.98			
C6	0.2	1.00	0.33	0.33	0.33	1	0.44	0.05	0.37	6.44			
							7.67						

Step 3: Now we compare alternatives with respect to each criteria to see how pair wise comparison matrix of alternatives are satisfying criteria. The comparison of one alternative to another for criteria C4 is shown for the demonstration purpose in Table 5. CR in this table is much less than 0.1, hence, there is good consistency of weight assigned by an expert. Similarly other alternative to alternative pair wise comparison matrix can be obtained. These are presented in Table 6.

Table 5: Pair-wise Comparison Matrix of Alternatives for Criteria C4

A1	At1	At2	At3	At4	At5	At6	GM	Weight (A1)	A3 =A1*A2	A4 =A3/A2	λ_{max}	CI	CR
At1	1	0.5	0.5	0.5	0.5	1	0.629	0.128	0.654	5.106	5.429	0.071	0.057
At2	0.5	1	0.5	1	0.5	2	0.793	0.161	0.924	5.723			
At3	1	1	1	0.5	2	1	1	0.203	1.109	5.449			
At4	1	0.5	0.5	1	1	0.5	0.707	0.143	0.726	5.049			
At5	1	0.5	1	2	1	0.5	0.890	0.181	0.972	5.362			
At6	0.5	1	2	0.5	1	1	0.890	0.181	1.067	5.886			
							4.912						

Table 6: Alternative to Alternative Weights						
Alternatives vs. Criteria	C1	C2	C3	C4	C5	C6
At1	0.180	0.163	0.177	0.128	0.143	0.152
At2	0.143	0.130	0.140	0.161	0.128	0.191
At3	0.202	0.115	0.158	0.203	0.181	0.214
At4	0.127	0.146	0.140	0.143	0.203	0.135
At5	0.202	0.260	0.223	0.181	0.181	0.170
At6	0.143	0.183	0.158	0.181	0.161	0.135

Step 4: AHP weights are obtained by multiplying relative normalized weights A2 from Table 4 with normalized data from Table 3 as shown below:

Table 7: AHP Weight						
Alternative	At1	At2	At3	At4	At5	At6
AHP Weight	0.51	073	0.73	0.61	0.80	0.47

(iv) Evaluation of Final Ranking Through TOPSIS: Various steps of TOPSIS as discussed above are followed one by one as mentioned below by utilizing the final weight matrix of Table 6 and Table 7.

Step 1 and 2: Normalized decision matrix and weighted normalized decision matrix are obtained as shown in Table 8 and 9 respectively.

Table 8: Normalized Decision Matrix						
AHP Weight	0.51	073	0.73	0.61	0.80	0.47
Alternatives vs. Criteria	C1	C2	C3	C4	C5	C6
At1	0.435	0.386	0.428	0.310	0.349	0.366
At2	0.345	0.306	0.340	0.391	0.310	0.462
At3	0.488	0.273	0.382	0.493	0.439	0.518
At4	0.307	0.344	0.340	0.348	0.493	0.326
At5	0.488	0.613	0.540	0.439	0.439	0.411
At6	0.345	0.433	0.382	0.439	0.391	0.326

Table 9: Weighted Normalized Decision Matrix						
WEIGHTS	0.55	0.91	0.81	0.61	0.9	0.53
Alternatives vs. Criteria	C1	C2	C3	C4	C5	C6
At1	0.222	0.284	0.314	0.189	0.279	0.173
At2	0.176	0.225	0.249	0.238	0.248	0.218
At3	0.250	0.201	0.280	0.300	0.351	0.245
At4	0.157	0.253	0.249	0.212	0.394	0.154
At5	0.250	0.451	0.396	0.268	0.351	0.195
At6	0.176	0.319	0.280	0.268	0.313	0.154

Step 3: PIS and NIS are calculated using equations 4 and 5 as presented in Table 10.

Table 10: Calculated Values of PIS and NIS						
Positive Ideal solution (vj*)	0.250	0.451	0.396	0.300	0.394	0.245
Negative Ideal solution (vj')	0.157	0.201	0.249	0.189	0.248	0.154

Steps 4 and 5: Once the separation measures (distance) from PIS and NIS are obtained using equations 6 and 7 respectively, the relative closeness (closeness coefficient) can be determined using equation 8 as presented in Table 11, which shows degree of satisfaction (Sun, 2010). Degree of satisfaction of INTC stock is 0.818 followed by HPQ (0.521), JNJ (0.411), IBM (0.349), CVX (0.344), and DIS (0.212). Stock with closeness coefficient (Wang and Chang, 2007) close to 1 has the shortest distance from PIS and the largest distance from NIS. In other words, we can say that a large closeness coefficient of a stock indicates better performance in the stock market and can be given higher priority in portfolio selection. This table also shows rank of stock based on value of C_i^*. Hence, stock INTC (A5) with highest weight value 0.818 is in first rank followed by HPQ(At3), JNJ(At6), IBM(At4), CVX(At1) and DIS(At2).

Table 11: Separation Measures and Relative Closeness Value with Final Ranking

Alternatives	S_i^*	S_i^-	C_i^*	TOPSIS Ranking
At1(CVX)	0.257	0.129	0.334	5
At2(DIS)	0.322	0.086	0.212	6
At3(HPQ)	0.279	0.304	0.521	2
At4(IBM)	0.292	0.156	0.349	4
At5(INTC)	0.074	0.333	0.818	1
At6(JNJ)	0.228	0.159	0.411	3

CONCLUSION

Stock portfolio selection is a complex financial problem in which we need to identify the best stock to allocate fund in appropriate portions so that the desired return is achieved for a given level of risk. Since the portfolio selection problem is a multi criteria decision-making process, ranking of the stocks must be determined based on profound techniques. This research work demonstrates how to integrate AHP and TOPSIS to find out the best ranking of the stocks with six different criteria. The AHP is used to find weights and the TOPSIS is used to find final ranking based on the weights obtained through AHP.

REFERENCES

Ehrgott, M., Klamroth, K., & Schwehm, C. (2004). An MCDM Approach to Portfolio Optimization. *European Journal of Operational Research*, 155(3), 752–770.

Hwang, C. L. and Masud, A. S. M. (1979). *Multiple Objective Decision Making: Methods and Applications.* Springer-Verlag, Berlin.

Hwang, C. L. and Yoon, K. (1981). *Multiple Attribute Decision Making Methods and Applications*. Springer, Berlin.

Lee, W. S., Tzeng, G. H., Guan, J. L., Chien, K. T., & Huang, J. M. (2009). Combined MCDM techniques for exploring stock selection based on Gordon model. *Expert Systems with Applications*, 36(3), 6421–6430.

Rao, R.V. (2007). *Decision making in the manufacturing environment*, Springer-Verlag London Limited.

Saaty, T. L. (1994). *Fundamentals of Decision Making and Priority Theory with the Analytical Hierarchy Process*, RWS Publications, Pittsburgh.

Saaty, T. L.(1980).The Analytical Hierarchy Process, McGraw Hill, New York.

Saaty, T.L., Rogers, P.C. and Bell, R. (1980). Portfolio selection through hierarchies. *The Journal of Portfolio Management*, 16–21.

Sharma, Hari P. and Sharma, Dinesh K. (2005). A Multi-objective Decision Making Approach to Mutual Fund Portfolio Selection Problems. *Journal of Business and Economics Research*, 3(20), 1-10.

Steuer, R. E. (2003). Multiple criteria decision making combined with finance: A categorized bibliographic study. *European Journal of Operational Research*, 150, 496-515.

(A complete list of references is available upon request from Dinesh K. Sharma at dksharma@umes.edu)