CLOSED LOOP SUPPLY CHAIN FOR DETERIORATING ITEMS WITH EXPONENTIAL DEMAND AND MULTIVARIATE REMANUFACTURING RATE

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ABSTRACT

In this paper, we have presented a closed loop supply chain model for decaying items. The model is developed with time dependent exponentially increasing demand rate and multivariate production and remanufacturing rates. We consider the production and remanufacturing rates, dependent on demand and inventory level. Buyback products are collected from the end user and the return rate is taken to be linearly time dependent. Mathematical formulation has been provided to find the optimal solutions for the prescribed model. Finally a numerical analysis and sensitivity analysis are presented to describe the situation.
Keywords: EPQ model, Reverse logistics, Deterioration, optimality

INTRODUCTION

Reverse logistics stands for all operations related to the recycle of products and materials. It is the process of planning, implementing, and organizing the efficient, cost effective flow of raw materials, in-process inventory. Anthology of used products, as paper, bottle, and battery, is a known idea in present economies. Repairing, remanufacturing and recycling of cars and electronic appliances, and disposal of perilous waste are very recent research fields. The listed activities include a very broad area, and it seems to have different management problems. We refer to the term "Reverse logistics" as all activity associated with a product/service after the point of sale, the ultimate target to optimize or make more efficient aftermarket activity, thus saving money.

FORMULATION OF THE GENERAL MODEL

The model under consideration is developed with the following assumptions and notations:
1. The demand rate is assumed to increase exponentially.
2. New items are produced at rate, where.
3. Returned items are collected at the rate which is the linear function of time and reproduced at a rate where.
4. The constant deterioration at the three stocks are respectively denoted by
5. There is no repair or replacement of deteriorated items
6. We shall require that

NOTATIONS

1. The cost parameters for the manufacturing stock are as follows	
$p_m =$	Unit cost, which includes materials cost.
$s_m =$	Unit manufacturing cost, which includes cost components such as labor, energy and machinery.
$h_m =$	Unit holding cost per unit per unit time.

$K_m =$	Set-up cost per cycle.
2. The cost parameters for the remanufacturing stock are as follows :	
$S_r =$	Unit remanufacturing cost.
$h_r =$	Unit holding cot per unit per unit time.
$K_r =$	Set-up cost per cycle.
3. The cost parameters for the returned stock as follows :	
$p_R =$	Unit cost, which includes purchase cost
$h_R =$	Unit holding cost per unit per unit time
$k_R =$	Order cost per cycle

For each reproduction cycle, $I_r(t)$ denotes the inventory level at time *t*. The system starts operating at time T_0 by which the reproduction process starts and the inventory level increases until time T_1, where the stock-level reaches its maximum, and the reproduction process stopped.

Then the stock level decreases continuously where it becomes zero by time T_2. The process is repeated. Similarly for each production cycle, $I_m(t)$ denotes the inventory level at time *t*. starting at time T_2, the inventory level increases until time T_3 where it reaches its maximum, and the production process stopped. Then the inventory level decrease continuously and becomes zero by time T_4. The process is repeated also for returned cycle, $I_R(t)$ denotes the inventory level at time *t*. The system starts by the time T_0, the inventory level, decreases until time T_1 and the inventory level becomes zero. Then inventory level increases until the time T_4 where, the inventory level reaches its maximum. The process is repeated. (Fig. 2)

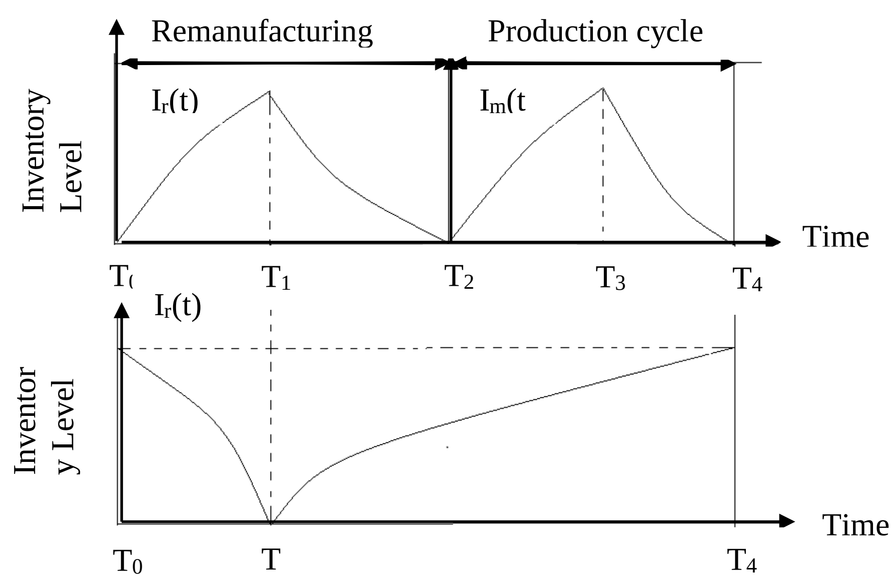

Fig. 1: Inventory variation of an EPQ model for Reverse logistics system.

The changes in the inventory levels depicted in fig. 2 are governed by the following differential equations:

$\frac{dI_r(t)}{dt}+(\delta_r+b_r)I_r(t)=a_r+(C_r-1)D(t),$	$T_0 \le t \le T_1$	$I_r(T_0)=0$	... (1)
$\frac{dI_r(t)}{dt}+\delta_r I_r(t)=-D(t),$	$T_1 \le t \le T_2$	$I_r(T_2)=0$	... (2)
$\frac{dI_m(t)}{dt}+(\delta_m+b_m)I_m(t)=a_m+(C_m-1)D(t),$	$T_2 \le t \le T_3$	$I_m(T_2)=0$	... (3)
$\frac{dI_m(t)}{dt}+(\delta_m)I_m(t)=-D(t),$	$T_3 \le t \le T_4$	$I_m(T_4)=0$	... (4)
$\frac{I_R(t)}{dt}+(\delta_R-b_r)I_r(t)=-a_r-C_rD(t)+R(t),$	$T_0 \le t \le T_1$	$I_R(T_1)=0$	... (5)
$\frac{I_R(t)}{dt}+\delta_R I_R(t)=R(t),$	$T_1 \le t \le T_4$	$I_R(T_1)=0$	... (6)

The solution of the above differential equations is

$I_r(t)=e^{-(\delta_r+b_r)t}\int_{T_0}^{t}\{a_r+(C_r-1)D(u)\}e^{(\delta_r+b_r)u}du,$	$T_0 \le t \le T_1$	... (7)
$I_r(t)=e^{-\delta_r t}\int_{t}^{T_2}D(u)e^{\delta_r u}du,$	$T_1 \le t \le T_2$	... (8)
$I_m(t)=e^{-(\delta_m+b_m)t}\int_{T_2}^{t}\{a_m+(C_m-1)D(u)\}e^{(\delta_m+b_m)u}du,$	$T_2 \le t \le T_3$	... (9)
$I_m(t)=e^{-\delta_m t}\int_{t}^{T_4}D(u)e^{\delta_m u}du,$	$T_3 \le t \le T_4$	... (10)
$I_R(t)=e^{-(\delta_R-b_r)t}\int_{t}^{T_1}\{a_r+C_rD(u)-R(u)\}e^{(\delta_R-b_r)u}du,$	$T_0 \le t \le T_1$	... (11)
$I_R(t)=e^{-\delta_R t}\int_{T_1}^{t}R(u)e^{\delta_R u}du,$	$T_1 \le t \le T_4$	... (12)

Respectively.

Let $I(t_1, t_2)=\int_{t_1}^{t_2}I(u)du,$ then from equation (7)-(12) we have

$$I_r(T_0,\,T_1)=\int_{T_0}^{T_1} e^{-(\delta_r+b_r)t}\int_{T_0}^{t}\{a_r+(C_r-1)\,D(u)\}\,e^{(\delta_r+b_r)u}du\,dt \qquad \text{... (13)}$$

$$I_r(T_1,\,T_2)=\int_{T_1}^{T_2} e^{-\delta_r t}\int_{t}^{T_2} D(u)\,e^{\delta_r u}dudt \qquad \text{... (14)}$$

$$I_m(T_2,\,T_3)=\int_{T_2}^{T_3} e^{-(\delta_m+b_m)t}\int_{T_2}^{t}\{a_m+(C_m-1)\,D(u)\}\,e^{(\delta_m+b_m)u}dudt \qquad \text{... (15)}$$

$$I_m(T_3,T_4)=\int_{T_3}^{T_4} e^{-\delta_m t}\int_{t}^{T_4} D(u)\,e^{\delta_m u}du\,dt \qquad \text{... (16)}$$

$$I_R(T_0,\,T_1)=\int_{T_0}^{T_1} e^{(\delta_R-b_r)t}\int_{t}^{T_1}\{a_r+C_rD(u)-R(u)\}\,e^{(\delta_R-b_r)u}du\,dt \qquad \text{... (17)}$$

$$I_R(T_1,\,T_4)=\int_{T_1}^{T_4} e^{-\delta_R t}\int_{T_1}^{t} R(u)e^{\delta_R u}du\,dt \qquad \text{... (18)}$$

Respectively.
Now using the integration by parts (13)-(18) are reduced to

$$I_r(T_0,\,T_1)=\frac{1}{(\delta_r+b_r)}\int_{T_0}^{T_1}\left\{e^{-(\delta_r+b_r)u}-e^{-(\delta_r+r)T_1}\right\}\{a_r+(C_r-1)D(u)\}\,e^{(\delta_r+b_r)u}du \qquad \text{... (19)}$$

$$I_r(T_1,\,T_2)=\frac{1}{\delta_r}\int_{T_1}^{T_2}\left(e^{-\delta_r T_1}-e^{-\delta_r u}\right)D(u)e^{\delta_r u}du \qquad \text{... (20)}$$

$$I_m(T_2,\,T_3)=\frac{1}{(\delta_m+b_m)}\int_{T_2}^{T_3}\left\{e^{-(\delta_m+b_m)u}-e^{-(\delta_m+b_m)T_3}\right\}\{a_m+(C_m-1)\,D(u)\}\,e^{(\delta_m+b_m)u}du$$
$$\text{... (21)}$$

$$I_m(T_3,\,T_4)=\frac{1}{\delta_m}\int_{T_3}^{T_4}\left\{e^{-\delta_m T_3}-e^{-\delta_m u}\right\}D(u)e^{\delta_m u}du \qquad \text{... (22)}$$

$$I_R(T_0,\,T_1)=\frac{1}{(\delta_R-b_r)}\int_{T_0}^{T_1}\left\{e^{-(\delta_R-b_r)T_0}-e^{-(\delta_R-b_r)u}\right\}\{a_r+C_rD(u)-R(u)\}\,e^{(\delta_R-b_r)u}du \qquad \text{... (23)}$$

$$I_R(T_1,\,T_4)=\frac{1}{\delta_R}\int_{T_1}^{T_4}\left\{e^{-\delta_R u}-e^{-\delta_R T_4}\right\}R(u).e^{\delta_R u}du \qquad \text{... (24)}$$

Note that we can set $T_0=0$ without loss of generality. Now the per cycle cost components for the given inventory system are as follows.

Production cost: $s_m\int_{T_2}^{T_3} P_m(u)du$

$$=s_m\left(\int_{T_2}^{T_3}\{a_m+C_mD(u)\}\,du+b_m\int_{T_2}^{T_3} I_m(u)\,du\right)$$

Remanufacturing cost $=s_r\int_0^{T_1} P_r(u)du$

$$=S_r\left(\int_0^{T_1}\left[a_r+C_rD(u)\right]du-b_r\int_0^{T_1} I_r(t)dt\right)$$

Holding cost $= h_r\left[I_r(0, T_1) + I_r(T_1, T_2)\right] + h_m\left[I_m(T_2, T_3) + I_m(T_3, T_4)\right]$

$+h_R\left[I_R(0, T_1) + I_R(T_1, T_4)\right]$

Thus, the total cost per unit time of the inventory system during the cycle $[0, T_4]$, as a function of T_1, T_2, T_3 and T_4 say $Z(T_1, T_2, T_3, T_4)$ is given by

$$Z(T_1, T_2, T_3, T_4) = \frac{1}{T_4}\Bigg\{S_r\int_0^{T_1}\left(a_r + C_r D(u)\right)du + (p_m + S_m)\int_{T_2}^{T_3}[a_m$$

$$+C_m D(u)]du + p_R\int_0^{T_4} R(u)du + \frac{\left[h_m - (p_m + s_m)b_m\right]}{(\delta_m + b_m)}$$

$$\int_{T_2}^{T_3}\left[1 - e^{(\delta_m + b_m)(u - T_3)}\right]\left[a_m + (C_m - 1)D(u)\right]du + \frac{h_m}{\delta_m}\int_{T_3}^{T_4}[e^{\delta_m(u - T_3)} - 1]D(u)du$$

$$du + \frac{[h_r - s_r b_r]}{(\delta_r + b_r)}\int_0^{T_1}\left[1 - e^{(\delta_r + b_r)(u - T_1)}\right]\left[a_r + (C_r - 1)D(u)\right]du$$

$$+\frac{h_r}{\delta_r}\int_{T_1}^{T_2}\left[e^{\delta_r(u - T_1)} - 1\right]D(u)du + \frac{h_R}{(\delta_R - b_r)}\Bigg[\int_0^{T_1}$$

$$\left[e^{(\delta_R - b_r)u} - 1\right]\left[a_r + C_r D(u) - R(u)\right]du\Bigg] + \frac{h_R}{\delta_R}\int_{T_1}^{T_4}\left(1 - e^{\delta_R(u - T_4)}\right)$$

$$R(u)du + K_r + K_m + K_R \Bigg\} \qquad \text{... (25)}$$

With the conditions

$$0 \le T_1 < T_2 < T_3 < T_4 \qquad \text{... (26)}$$

$$e^{-(\delta_r + b_r)T_1}\int_0^{T_1}\left(a_r + (C_r - 1)D(u)\right)e^{(\delta_r + b_r)u}du$$

$$= e^{-\delta_r T_1}\int_{T_1}^{T_2} D(u)e^{\delta_r u}du \qquad \text{... (27)}$$

$$e^{-(\delta_m + b_m)T_3}\int_{T_2}^{T_3}\left[a_m + (C_m - 1)D(u)\right]e^{(\delta_m + b_m)u}du$$

$$= e^{-\delta_m T_3} \int_{T_3}^{T_4} D(u) e^{\delta_m u} du \qquad \text{... (28)}$$

$$\int_0^{T_1} \left[a_r + C_r D(u) - R(u)\right] e^{(\delta_R - b_r)u} du$$

$$= e^{-\delta_R T_4} \int_{T_1}^{T_4} R(u) e^{\delta_R u} du \qquad \text{... (29)}$$

CONCLUSION

In this paper a general reverse logistics inventory model for integrated production of new items and reproduction of returned items is presented. Examples, which explain the application of the theoretical results and these illustrative examples are numerically verified too. Further, the effects of demand, different deterioration and return rates are compared. In future study the extension is to assume that the production and reproduction rates follow learning and forgetting curves. The second extension is to study the case where shortages are allowed in each production and reproduction cycle. A third extension is to consider multiple production and reproduction batches per interval.

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