NON -INSTANTANEOUS DETERIORATION INVENTORY MODEL WITH PRICE AND STOCK DEPENDENT DEMAND UNDER INFLATION IN FUZZY ENVIRONMENT

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ABSTRACT

In this paper we have developed a fuzzy inventory model with the assumption of price and stock dependent demand, fully backlogged shortages and inflation. In fuzzy model, we considered triangular numbers. In this model we have described the demand function is dependent on price and stock and in shortage time the demand is depend only price of the product. The non-instantaneous deterioration is inventory problem constitutes a constraint optimization problem. Here we have solved this problem by using MATHEMATICA software. Finally, illustrate and validate the inventory model, we have used a numerical example considered different rate. A sensitivity analysis has been carried out to study the effect of different inventory parameters changing one parameter at a time and the others value of parameters is same.

Keywords: Fully backlogged Shortages; Inflation; Non-instantaneous Deterioration, Stock and Price Dependent Demand; Fuzzy.

INTRODUCTION

A lot of researchers/scientists have been presented different types inventory models considered the product as a deteriorating item in the existing literature. The deterioration is one of the important factors in inventory analysis in real life situation. However, it is affecting the inventory system because deteriorating product is not useable, i.e., it is fully damage. So we cannot neglect this effect in inventory analysis. This type of model was first introduced by Ghare and Schrader (1963), He has developed an inventory model considered deterioration factor and deterioration rate is constant. Ghare and Schrader (1963) model was extended by Covert and Philip (1973) considered the deterioration rate follows a two-parameter Weibull distribution. Dave and Patel (1981) discussed an EOQ (economic order quantity) inventory model for deteriorating items with time-dependent demand without shortages. Another important factor is shortage considered by Sachan (1984) which is extension of Dave and Patel (1981). In this relation we may refer several correlated research work was introduced by Wee (1995), Hariga (1996), Wee and Law (1999), Moon et al. (2005), Chung and Wee (2007), Sana (2010a), Bhunia and Shaikh (2011a, 2011b, 2014, 2015, 2016), Widyadana et al. (2011), Bhunia et al. (2013, 2014, 2015a, 2015b, 2015c, 2016), Sarkar et al. (2013), Shaikh (2016 a, 2016b) and others.

Inflation is also another important factor in inventory modelling. In inventory analysis many researchers accepted and discussed about this factor and its impact on the inventory analysis. Buzacott (1975) have presented inventory models and considered the inflation effect for all the related costs. Bierman and Thomas (1977) have proposed an EOQ model considering the effect of inflation. Then, Misra (1979) has developed an inventory model with different inflation rates for different inventory costs. After that, Padmanabhan and Vrat (1990) have introduced an EOQ model for stock-dependent demand and exponential deterioration. Thereafter Datta and Pal (1991) have presented an inventory model with demand which depends on linearly in time and shortages by considering the effects of inflation. Wee and Law (1999, 2001) proposed deteriorating inventory models considered the inflation effect. Some related works such as Yang (2004, 2006, 2012), Jaggiet al. (2006, 2011), Hsieh et al. (2008), Yang and Chang

(2013), Bhunia et al. (2015a, 2016) and others also contributed in this field of research. In the aforementioned works, the authors have been considered deterioration and inflation effect. Giri et al. (2003) developed an inventory model with shortage, ramp-type demand rate and the time dependent deterioration rate. Manna and Chaudhuri (2006) have been introduced an EOQ model for deteriorating items with time-dependent demand. Loa et al. (2007) have been developed an integrated production model for Weibull distribution deterioration under inflation and imperfect production processes. An inventory model with Weibull deterioration rate, ramp type demand rate and partial backlogging was considered by Skouri et al. (2009). Sana (2010b) have been developed aneconomic order quantity model with partial backlogging rate and time varying deterioration. Sarkar (2012a) have been developed an EOQ model considered the demand and deterioration rate were both dependent on time. Sett et al. (2012) have been introduced two-warehouse inventory model with time dependent deterioration and quadratic demand. Sarkar (2012b) have been presented a production-inventory model considered three different types of continuously distributed deterioration functions. Sarkar and Sarkar (2013) have been introduced a non-instantaneous deteriorating inventory model with stock dependent demand and partial backlogged shortage.

In this work we have presented an inventory model for non-instantaneous deteriorating item with price, stock dependent, fully backlogged shortage under inflation and fuzzy environment. The demand function is price and stock dependent in without shortage period and when shortage appear then demand is only price dependent. In this work, we have considered the deterioration is a non-instantaneous. Shortages, if any are allowed and it is fully backlogged. In this work, we have solved this problem by using MATHEMATICA software. Finally, to illustrate and validate the proposed inventory model, we have used a numerical example considered different fixed markup rate. A sensitivity analysis has also been performed to study the effect of changes of different inventory parameters changing one parameter at a time and the others value of parameters is same.

ASSUMPTIONS

1. Different demand rate is considered in this paper

$$D(p)=\begin{cases} a-bp+\beta I(t) & I(t)>0 \\ a-bp & I(t)<0 \end{cases}$$

Demand is selling price as well as stock dependent when $I(t) > 0$ and demand is price dependent when $I(t) < 0$.

2.Shortages are considered as fully backlogged.

3.Replenishment rate is instantaneous.

4.Lead time is negligible.

5.Infinite planning horizon is considered.

6.Inflation is also taken in this paper with rate r.

7.This paper is solved in both crisp and fuzzy environment.

NOTATIONS

C_0	Replenishment cost per order $/unit
C_p	Purchasing cost per unit $/unit
C_h	Holding cost per unit per unit time $/unit
Cs	Shortage cost per unit per unit time $/unit
θ	Constant deterioration rate
S	Maximum stock per cycle in units
p	Selling price and $p = mc_p$ $/unit
m	Fixed markup rate
R	maximum shortage level in units
I(t)	Inventory level at any time t where $0 \le t \le T$ in units
$TC(t, t_1)$	The total cost per unit time $/year
r	Inflation rate

MATHEMATICAL MODEL FORMULATION FOR INVENTORY MODEL

We have developed an inventory model based with above-mentioned assumptions. Initially an endeavour purchased of goods (S + R) units. This stock down due to meet up the customers' demands as well as deterioration. At time $t = t_1$ stock will be zero. After that shortage emerges. Therefore, the inventory system describes by the following differential equations:

$$\frac{dI_1(t)}{dt} = -(a-bp) - \beta I_1(t), \qquad 0 < t \le t_d \tag{1}$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(a-bp) - \beta I_2(t), \qquad 0 < t \le t_d \tag{2}$$

$$\frac{dI_3(t)}{dt} = -(a-bp), \qquad t_1 < t < T \tag{3}$$

On solving these differential equations (1) – (3) with the boundary conditions $I(t) = S$, $I(t) = -R$, at $t = 0$ and $t = T$ and I(t) is continuous at $t = t_d$ and $t = t_1$. Hence from equation (1)- (3) we have

$$I_1(t) = Se^{-\beta t} + \frac{(a-bp)}{\beta}\{e^{-\beta t} - 1\}, \qquad 0 < t \le t_d \tag{4}$$

Using the condition $I(t) = S$ at $t = 0$, we have obtain

$$S = \frac{a-bp}{\theta+\beta}\{e^{(\theta+\beta)t_1} - 1\} \tag{5}$$

$$I_2(t) = \frac{a-bp}{\theta+\beta}\{e^{(\theta+\beta)(t_1-t)} - 1\} \tag{6}$$

From equation (3), we have

$$I_3(t) = (a-bp)(t_1 - t) \tag{7}$$

From equation (6), using the condition $I(t) = -R$ at $t = T$, we get

$$R = (a-bp)(T - t_1) \tag{8}$$

Total cost per unit time for the inventory system consists of the following components:

(a) Ordering cost per cycle = $C_0 e^{-rT}$ (9)

(b) The inventory holding cost per cycle = $C_h\left[\int_0^{t_d} e^{-rt} I_1(t)dt + \int_{t_d}^{t_1} e^{-rt} I_2(t)dt\right]$

$$= C_h\left[\frac{-S}{(\beta+r)}\{e^{-(\beta+r)t_d} - 1\} - \frac{(a-bp)}{\beta(\beta+r)}\{e^{-(\beta+r)t_d} - 1\} + \frac{(a-bp)}{\beta r}\{e^{-rt_d} - 1\} + \frac{(a-bp)}{(\beta+\theta)r}\{e^{-rt_1} - e^{-rt_d}\} - \frac{(a-bp)}{(\beta+\theta)(\beta+\theta+r)} e^{(\theta+\beta)t_1}\{e^{-(\theta+\beta+r)t_1} - e^{(\theta+\beta+r)t_d}\}\right] \tag{10}$$

(c) Purchase cost pe4r cycle = C_p (S+R) (11)

(d) Shortage cost = $C_s \int_{t_1}^{T} -e^{-rt} I_3(t)dt$

$$= C_s(a-bp)\left[\frac{t_1 e^{-rT}}{r} - \frac{Te^{-rT}}{r} + \frac{1}{r^2}\{e^{-rt_1} - e^{-rT}\}\right] \tag{12}$$

Total Inventory Average Cost = Ordering cost + Holding Cost + Purchase cost + Shortage Cost

i.e., TC = $\frac{1}{T}$ [C_0. OC + C_h. HC + C_p. PC + C_s. SC] (13)

FUZZY MODEL

To develop the model in a fuzzy environment, we consider the costs as the triangular fuzzy numbers where, $C_0 = (C_0 - \Delta_1, C_0, C_0 + \Delta_2)$, $C_h = (C_h - \Delta_3, C_h, C_h + \Delta_4)$, $C_p = (C_p - \Delta_5, C_p, C_p + \Delta_6)$, $C_s = (C_s - \Delta_7, C_s, C_s + \Delta_8)$ such that $0 < \Delta_1 < C_0$, $0 < \Delta_2$, $0 < \Delta_3 < C_h$, $0 < \Delta_4$, $0 < \Delta_5 < C_p$, $0 < \Delta_6$, $0 < \Delta_7 < C_s$, $0 < \Delta_8$ and $\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7, \Delta_8$ are determined by the decision maker based on the uncertainty of the problem. Thus, the costs are considered as the fuzzy numbers with membership function

$$TC = \frac{1}{T}[\tilde{C}_0.OC + \tilde{C}_h.HC + \tilde{C}_p.PC + \tilde{C}_s.SC] \tag{14}$$

By Centroid Method, we get

$$\tilde{C}_0 = C_0 + \frac{1}{3}(\Delta_2 - \Delta_1) \tag{15}$$

$$\tilde{C}_h = C_h + \frac{1}{3}(\Delta_4 - \Delta_3) \tag{16}$$

$$\tilde{C}_p = C_p + \frac{1}{3}(\Delta_6 - \Delta_5) \tag{17}$$

$$\tilde{C}_s = C_s + \frac{1}{3}(\Delta_8 - \Delta_7) \tag{18}$$

$$F_1 = \frac{1}{T}[(C_0 + \Delta_2).OC + (C_h + \Delta_4).HC + (C_p + \Delta_6).PC + (C_s + \Delta_8).SC] \tag{19}$$

$$F_2 = \frac{1}{T}[C_0.OC + C_h.HC + C_p.PC + C_s.SC] \tag{20}$$

$$F_3 = \frac{1}{T}[(C_0 - \Delta_1).OC + (C_h - \Delta_3).HC + (C_p - \Delta_5).PC + (C_s - \Delta_7).SC] \tag{21}$$

By the method of Defuzzification, we have

$$= \frac{1}{T}[\{C_0 - \frac{(\Delta_2 - \Delta_1)}{4}\}.OC + \{C_h - \frac{(\Delta_4 - \Delta_3)}{4}\}.HC + \{C_p - \frac{(\Delta_6 - \Delta_5)}{4}\}.PC + \{C_s - \frac{(\Delta_8 - \Delta_7)}{4}\}.SC] \tag{22}$$

The above mention problems can be solved by using the software MATHEMATICA 9.0 and the following algorithm.

SOLUTION PROCEDURE

We have solved the above-mentioned problem using the following algorithm:
Step 1 Input the value of all required parameters of the proposed inventory model.
Step 2 Solve the above-discussed constrained optimisation problem and store the optimal value of TC^*, S^*, R^*, $t_1{*}$ T^*.
Step 3 Stop.

NUMERICAL ILLUSTRATIONS

To demonstrate the developed model, a numerical example with the following values of different parameters has been considered. Let C_h= \$2.5 per unit per unit time, C_o= \$500 per order, C_s= \$8.00 per unit per unit time, C_p= \$12.00 per unit, a = 100, b = 0.4, θ = 0.065, β = 0.50, r = 0.03, t_d= 1.8. The parameter values taken here are realistic, thought these values are not considered from any case study of an existing inventory system.

m	S	R	t_1	T	TC
1.20	90.9862	32.4788	0.565567	1.00067	941.78
1.25	90.6966	32.5678	0.565567	1.00098	634.71
1.30	90.3558	32.7553	0.565568	1.00121	592.82
1.35	90.1177	32.9633	0.565567	1.00323	288.75

OBSERVATIONS

1. Total cost (TC) is a decreasing function when markup rate (m) increases.
2. Stock level (S) is decreasing when markup rate (m) increases.
3. Shortage level is increasing when markup rate (m) increases.
4. The total average cost is highly sensitive w.r.t. the demand parameters *a* and purchase cost C_p.
5.Cycle length of the system is highly sensitive w.r.t.C_p. It is moderatelysensitive with respect to other parameters.
6. The highest shortage level is highly sensitive with respect to $C_s$and insensitive w.r.t the other parameters.
7. The on-hand stock-level *S* is highly sensitive w.r.t. the parameter *a*,*b* and C_p.

CONCLUDING REMARKS

In this work, we have presented an inventory model with the consideration of price and stock dependent demand, fully backlogged shortage and inflation, fuzzy environment. Firstly demand function is dependent on price and stock and when shortage appears, demand is depending only price of the item. Price of the item is dependent of different markup rate. In this paper, we have also showed the effect of inflation in the whole inventory system. The deterioration is considered as a non-instantaneous, Shortages, if any are allowed and it is fully backlogged. This model is presented in both crisp and fuzzy environment. For further research, one can extend the proposed model in several ways. This model can be extended for different types of variable demand dependent on displayed stock-level, time dependent demand, finite time horizon, price and time dependent demand and single level trade credit. On the other hand, it can also be generalised by considering two level credit policies.

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